1 MECHANISMS AND MACHINES

Introduction

If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*. A mechanism transmits and modifies a motion. A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. Thus, a mechanism is a fundamental unit and one has to start with its study.

The study of a mechanism involves its analysis as well as synthesis. *Analysis* is the study of motions and forces concerning different parts of an existing mechanism, whereas *synthesis* involves the design of its different parts. In a mechanism, the various parts are so proportioned and related that the motion of one imparts requisite motions to the others and the parts are able to withstand the forces impressed upon them. However, the study of the relative motions of the parts does not depend on the strength and the actual shapes of the parts.

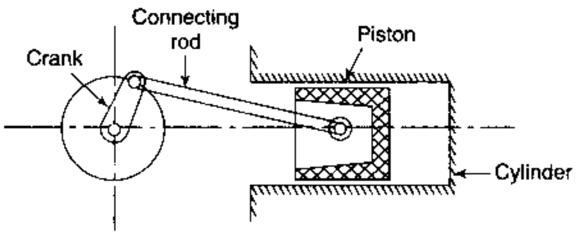

Fig. 1.1

In a reciprocating engine, the displacement of the piston depends upon the lengths of the connecting rod and the crank (Fig. 1.1). It is independent of the bearing strength of the parts or whether they are able to withstand the forces or not. Thus for the study of motions, it is immaterial if a machine part is made of mild steel, cast iron or wood. Also, it is not necessary to know the actual shape and area of the cross section of the part. Thus, for the study of motions of different parts of a mechanism, the study of forces is not necessary and can be neglected. The study of mechanisms, therefore, can be divided into the following disciplines:

Kinematics It deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions. Thus, it is the study, from a geometric point of view, to know the displacement, velocity and acceleration of a part of a mechanism.

Dynamics It involves the calculations of forces impressed upon different parts of a mechanism. The forces can be either static or dynamic. Dynamics is further subdivided into *kinetics* and *statics*. Kinetics is the study of forces when the body is in motion whereas statics deals with forces when the body is stationary.

1.1 MECHANISM AND MACHINE

As mentioned earlier, a combination of a number of bodies (usually rigid) assembled in such a way that the motion of one causes constrained and predictable motion to the others is known as a *mechanism*. Thus, the function of a mechanism is to transmit and modify a motion.

A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. It is neither a source of energy nor a producer of work but helps in proper utilization of the same. The motive power has to be derived from external sources.

A slider-crank mechanism (Fig. 1.2) converts the reciprocating motion of a slider into rotary motion of the crank or vice-versa. However, when it is used as an automobile engine by adding valve mechanism, etc., it becomes a machine which converts the available energy (force on the piston) into the desired energy (torque of the crank-shaft). The torque is used to move a vehicle. Reciprocating pumps, reciprocating compressors and steam engines are other examples of machines derived from the slider-crank mechanism.

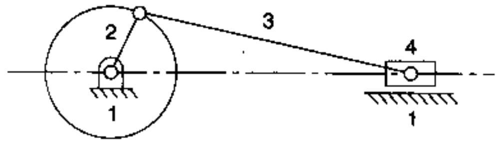

Fig. 1.2

Some other examples of mechanisms are typewriters, clocks, watches, spring toys, etc. In each of these, the force or energy provided is not more than what is required to overcome the friction of the parts and which is utilized just to get the desired motion of the mechanism and not to obtain any useful work.

1.2 TYPES OF CONSTRAINED MOTION

There are three types of constrained motion:

(i) ***Completely constrained motion*** When the motion between two elements of a pair is in a definite direction irrespective of the direction of the force applied, it is known as completely constrained motion. The constrained motion may be linear or rotary. The sliding pair of Fig. 1.3(a) and the turning pair of Fig. 1.3(b) are the examples of the completely constrained motion. In sliding pair, the inner prism can only slide inside the hollow prism. In case of a turning pair, the inner shaft can have only rotary motion due to collars at the ends. In each case the force has to be applied in a particular direction for the required motion.

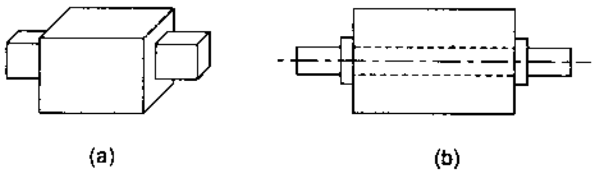

Fig. 1.3

(ii) ***Incompletely constrained motion*** When the motion between two elements of a pair is possible in more than one direction and depends upon the direction of the force applied, it is known as incompletely constrained motion. For example, if the turning pair of Fig. 1.4 does not have collars, the inner shaft may have sliding or rotary motion depending upon the direction of the force applied. Each motion is independent of the other.

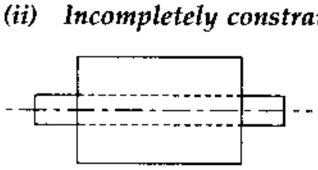

Fig. 1.4

(iii) ***Successfully constrained motion*** When the motion between two elements of a pair is possible in more than one direction but is made to have motion only in one direction by using some external means, it is a successfully constrained motion. For example, a shaft in a footstep bearing may have vertical motion apart from rotary motion (Fig. 1.5). But due to load applied on the shaft it is constrained to move in

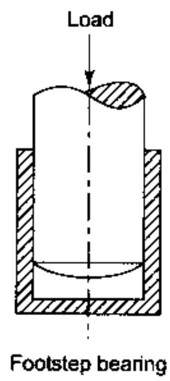

Fig. 1.5

that direction and thus is a successfully constrained motion. Similarly, a piston in a cylinder of an internal combustion engine is made to have only reciprocating motion and no rotary motion due to constrain of the piston pin. Also, the valve of an IC engine is kept on the seat by the force of a spring and thus has successfully constrained motion.

1.3 RIGID AND RESISTANT BODIES

A body is said to be *rigid* if under the action of forces, it does not suffer any distortion or the distance between any two points on it remains constant.

Resistant bodies are those which are rigid for the purposes they have to serve. Apart from rigid bodies, there are some semi-rigid bodies which are normally flexible, but under certain loading conditions act as rigid bodies for the limited purpose and thus are resistant bodies. A belt is rigid when subjected to tensile forces. Therefore, the belt-drive acts as a resistant body. Similarly, fluids can also act as resistant bodies when compressed as in case of a hydraulic press. For some purposes, springs are also resistant bodies.

These days, resistant bodies are usually referred as rigid bodies.

1.4 LINK

A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a *link*. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them. Thus, a link may consist of one or more resistant bodies. A slider-crank mechanism consists of four links: frame and guides, crank, connecting-rod and slider. However, the frame may consist of bearings for the crankshaft. The crank link may have a crankshaft and flywheel also, forming one link having no relative motion of these.

A link is also known as *kinematic link* or *element*.

Links can be classified into *binary*, *ternary* and *quaternary* depending upon their ends on which revolute or turning pairs (Sec. 1.5) can be placed. The links shown in Fig. 1.6 are rigid links and there is no relative motion between the joints within the link.

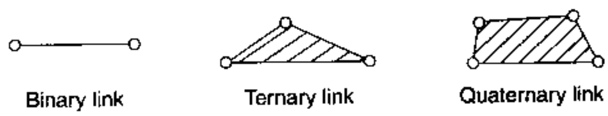

Fig. 1.6

1.5 KINEMATIC PAIR

A kinematic pair or simply a pair is a joint of two links having relative motion between them. In a slider-crank mechanism (Fig. 1.2), the link 2 rotates relative to the link 1 and constitutes a revolute or turning pair. Similarly, links 2, 3 and 3, 4 constitute turning pairs. Link 4 (slider) reciprocates relative to the link 1 and is a sliding pair.

Types of Kinematic Pairs Kinematic pairs can be classified according to

- nature of contact
- nature of mechanical constraint
- nature of relative motion

Kinematic Pairs according to Nature of Contact

(a) Lower Pair A pair of links having surface or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar.

Examples Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint, etc.

(b) Higher Pair When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

Examples Wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings, etc.

Kinematic Pairs according to Nature of Mechanical Constraint

(a) Closed Pair When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

All the lower pairs and some of the higher pairs are closed pairs. A cam and follower pair (higher pair) shown in Fig. 1.7(a) and a screw pair (lower pair) belong to the closed pair category.

(b) Unclosed Pair When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g., cam and follower pair of Fig. 1.7(b).

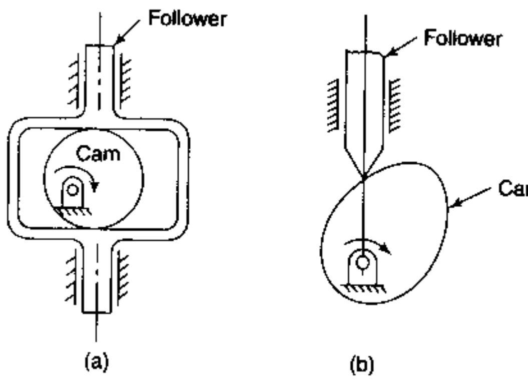

Fig. 1.7

Kinematic Pairs according to Nature of Relative Motion

(a) Sliding Pair If two links have a sliding motion relative to each other, they form a sliding pair.

A rectangular rod in a rectangular hole in a prism is a sliding pair [Fig. 1.8(a)].

(b) Turning Pair When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair [Fig. 1.8(b)].

In a slider-crank mechanism, all pairs except the slider and guide pair are turning pairs. A circular shaft revolving inside a bearing is a turning pair.

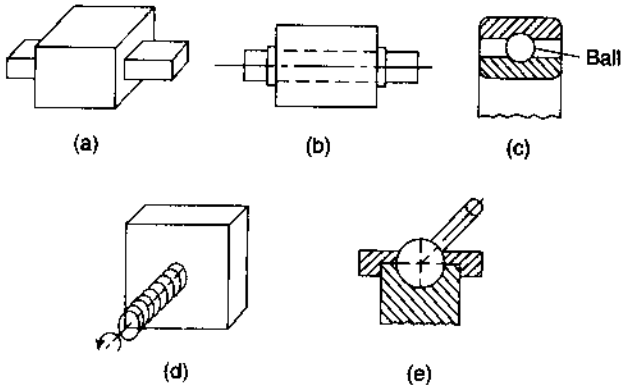

Fig. 1.8

(c) Rolling Pair When the links of a pair have a rolling motion relative to each other, they form a rolling pair, e.g., a rolling wheel on a flat surface, ball and roller bearings, etc. In a ball bearing [Fig. 1.8(c)], the ball and the shaft constitute one rolling pair whereas the ball and the bearing is the second rolling pair.

(d) Screw Pair (Helical Pair) If two mating links have a turning as well as sliding motion between them, they from a screw pair. This is achieved by cutting matching threads on the two links.

The lead screw and the nut of a lathe is a screw pair [Fig. 1.8(d)].

(e) Spherical Pair When one link in the form of a sphere turns inside a fixed link, it is a spherical pair.

The ball and socket joint is a spherical pair [Fig. 1.8(e)].

1.6 TYPES OF JOINTS

The usual types of joints in a chain are

- Binary joint
- Ternary joint
- Quaternary joint

Binary Joint If two links are joined at the same connection, it is called a binary joint. For example, Fig. 1.9 shows a chain with two binary joints named *B*.

Ternary Joint If three links are joined at a connection, it is known as a ternary joint. It is considered equivalent to two binary joints since fixing of any one link constitutes two binary joints with each of the other two links. In Fig. 1.9 ternary links are mentioned as *T*.

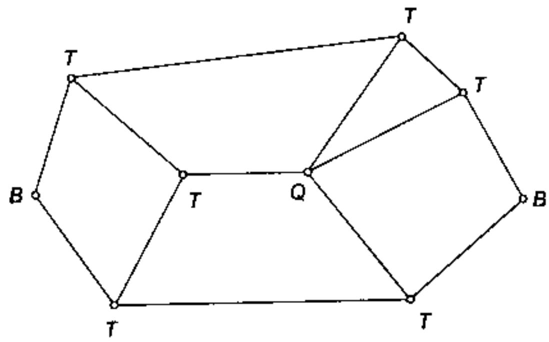

Fig. 1.9

Quaternary Joint If four links are joined at a connection, it is known as a quaternary joint. It is considered equivalent to three binary joints since fixing of any one link constitutes three binary joints. Figure 1.9 shows one quaternary joint.

In general, if n number of links are connected at a joint, it is equivalent to $(n - 1)$ binary joints.

1.7 DEGREES OF FREEDOM

An unconstrained rigid body moving in space can describe the following independent motions (Fig. 1.10):

1. Translational motions along any three mutually perpendicular axes x, y and z
2. Rotational motions about these axes

Thus, a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

Degrees of freedom = 6 – Number of restraints

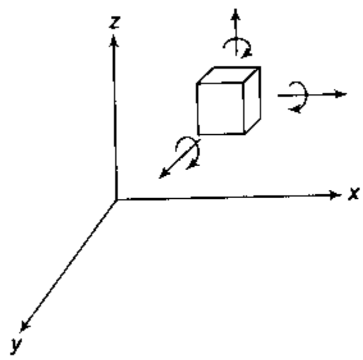

Fig. 1.10

1.8 CLASSIFICATION OF KINEMATIC PAIRS

Depending upon the number of restraints imposed on the relative motion of the two links connected together, a pair can be classified as given in Table 1.1 which gives the possible form of each class.

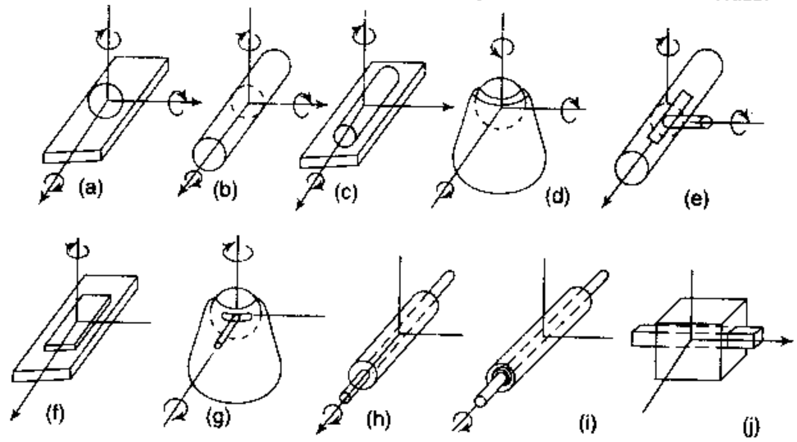

Fig. 1.11

Different forms of each class have also been shown in Fig. 1.11. Remember that a particular relative motion between two links of a pair must be independent of the other relative motions that the pair can have. A screw and nut pair permits translational and rotational motions. However, as the two motions cannot be accomplished independently, a screw and nut pair is a kinematic pair of the fifth class and not of the fourth class.

1.9 KINEMATIC CHAIN

A *kinematic chain* is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite [Fig.1.12 (a), (b), and (c)].

Table 1.1

Class	Number of Restraints	Form	Restraints on		Kinematic pair	Fig. 1.11
			Translatory motion	Rotary motion		
I	1	1st	1	0	Sphere-plane	a
II	2	1st	2	0	Sphere-cylinder	b
		2nd	1	1	Cylinder-plane	c
III	3	1st	3	0	Spheric	d
		2nd	2	1	Sphere-slotted cylinder	e
		3rd	1	2	Prism-plane	f
IV	4	1st	3	1	Slotted-spheric	g
		2nd	2	2	Cylinder	h
V	5	1st	3	2	Cylinder (collared)	i
		2nd	2	3	Prismatic	j

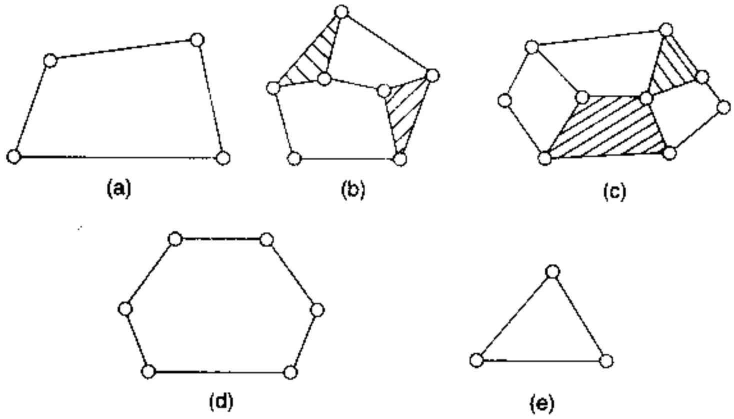

Fig. 1.12

In case the motion of a link results in indefinite motions of other links, it is a *non-kinematic chain* [Fig. 1.12(d)]. However, some authors prefer to call all chains having relative motions of the links as kinematic chains.

A *redundant chain* does not allow any motion of a link relative to the other [Fig. 1.12(e)].

1.10 LINKAGE, MECHANISM AND STRUCTURE

A *linkage* is obtained if one of the links of a kinematic chain is fixed to the ground. If motion of any of the moveable links results in definite motions of the others, the linkage is known as a *mechanism*. However, this distinction between a mechanism and a linkage is hardly followed and each can be referred in place of the other.

If one of the links of a redundant chain is fixed, it is known as a *structure* or *a locked system*. To obtain constrained or definite motions of some of the links of a linkage (or mechanism), it is necessary to know how many inputs are needed. In some mechanisms, only one input is necessary that determines the motions of other links and it is said to have one degree of freedom. In other mechanisms, two inputs may be necessary to get constrained motions of the other links and they are said to have two degrees of freedom, and so on.

The degree of freedom of a structure or a locked system is zero. A structure with negative degree of freedom is known as a *superstructure*.

1.11 MOBILITY OF MECHANISMS

A mechanism may consist of a number of pairs belonging to different classes having different number of restraints. It is also possible that some of the restraints imposed on the individual links are common or general to all the links of the mechanism. According to the number of these general or common restraints, a mechanism may be classified into a different order. A zero-order mechanism will have no such general restrain. Of course, some of the pairs may have individual restraints. A first-order mechanism has one general restraint; a second-order mechanism has two general restraints, and so on, up to the fifth order. A sixth-order mechanism cannot exist since all the links become stationary and no movement is possible.

Expressing the number of degrees of freedom of a linkage in terms of the number of links and the number of pair connections of different types is known as *number synthesis*. *Degrees of freedom* of a mechanism in space can be determined as follows:

Let

N = total number of links in a mechanism

F = degrees of freedom

P_1 = number of pairs having one degree of freedom

P_2 = number of pairs having two degrees of freedom, and so on

In a mechanism, one link is fixed.

Therefore,

Number of movable links = $N - 1$

Number of degrees of freedom of $(N - 1)$ movable links = $6(N - 1)$

Each pair having one degree of freedom imposes 5 restraints on the mechanism, reducing its degrees of freedom by $5P_1$.

Each pair having two degrees of freedom will impose 4 restraints, reducing the degrees of freedom of the mechanism by $4P_2$.

Similarly, other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of the mechanism. Thus,

$$F = 6(N - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 \quad (1.1)$$

The above criterion is hardly necessary to find the degrees of freedom, as space mechanisms, especially of the zero order are not practical. Most of the mechanisms are two-dimensional such as a four-link or a slider-crank mechanism in which displacement is possible along two axes (one restraint) and rotation about only one axis (two restraints). Thus, there are three general restraints.

Therefore, for plane mechanisms, the following relation may be used to find the degrees of freedom

$$F = 3(N - 1) - 2P_1 - 1P_2 \quad (1.2)$$

This is known as *Gruebler's criterion* for degrees of freedom of plane mechanisms in which each movable link possesses three degrees of freedom. Each pair with one degree of freedom imposes two further restraints on the mechanisms, thus reducing its degrees of freedom. Similarly, each pair with two degrees of freedom reduces the degrees of freedom of the mechanism at the rate of one restraint each.

Some authors mention the above relation as *Kutzback's criterion* and a simplified relation $[F = 3(N-1) - 2P_1]$ which is applicable to linkages with a single degree of freedom only as Gruebler's criterion. However, many authors make no distinction between Kutzback's criterion and Gruebler's criterion.

Thus, for linkages with a single degree of freedom only, $P_2 = 0$

$$F = 3(N - 1) - 2P_1 \quad (1.3)$$

Most of the linkages are expected to have one degree of freedom so that with one input to any of the links, a constrained motion of the others is obtained.

Then,

$$1 = 3(N - 1) - 2P_1$$

or

$$2P_1 = 3N - 4 \quad (1.4)$$

As P_1 and N are to be whole numbers, the relation can be satisfied only if N is even. For possible linkages made of binary links only,

$N = 4$,	$P_1 = 4$	No excess turning pair
$N = 6$,	$P_1 = 7$	One excess turning pair
$N = 8$,	$P_1 = 10$	Two excess turning pairs

and so on.

Thus, with the increase in the number of links, the number of excess turning pairs goes on increasing. Getting the required number of turning pairs from the required number of binary links is not possible. Therefore, the excess or the additional pairs or joints can be obtained only from the links having more than two joining points, i.e., ternary or quaternary links, etc.

For a six-link chain, some of the possible types are Watts six-bar chain, in which the ternary links are directly connected [Fig. 1.13(a)] and Stephenson's six-bar chain, in which ternary links are not directly connected [Fig. 1.13(b)]. Another possibility is also shown in Fig. 1.13(c). However, this chain is not a six-link chain but a four-link chain as links 1, 2 and 3 are, in fact, one link only with no relative motion of these links.

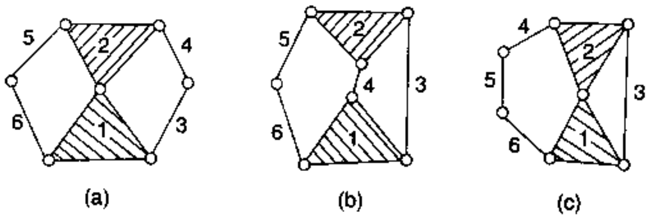

Fig. 1.13

Two excess turning pairs required for an eight-link chain can be obtained by using (apart from binary links):

four ternary links [Figs 1.14(a) and (b)]

two quaternary links [Fig. 1.14(c)]

one quaternary and two ternary links [Fig. 1.14(d)].

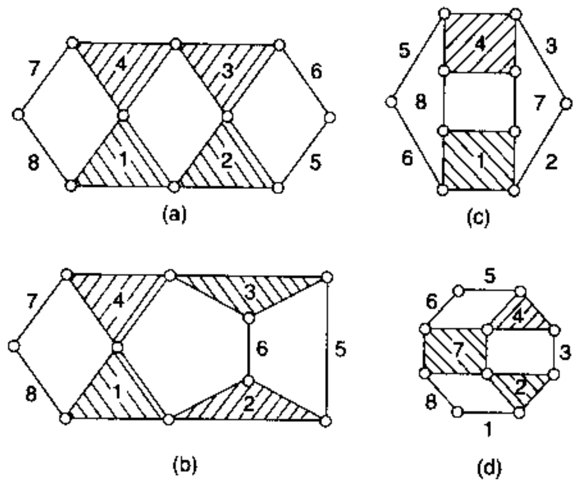

Fig. 1.14

Now, consider the kinematic chain shown in Fig. 1.15. It has 8 links, but only three ternary links. However, the links 6, 7 and 8 constitute a double pair so that the total number of pairs is again 10. The degree of freedom of such a linkage will be

$$F = 3\,(8 - 1) - 2 \times 10$$
$$= 1$$

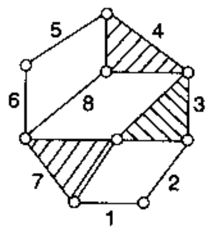

Fig. 1.15

This shows that the number of ternary or quaternary links in a chain can be reduced by providing double joints also.

The following empirical relations formulated by the author provide the degree of freedom and the number of joints in a linkage when the number of links and the number of loops in a kinematic chain are known. These relations are valid for linkages with turning pairs,

$$F = N - (2L + 1) \tag{1.5}$$

$$P_1 = N + (L - 1) \tag{1.6}$$

where

L = number of loops in a linkage.

Thus, for different number of loops in a linkage, the degrees of freedom and the number of pairs are as shown in Table 1.2.

Table 1.2

L	F	P_1
1	$N-3$	N
2	$N-5$	$N+1$
3	$N-7$	$N+2$
4	$N-9$	$N+3$
5	$N-11$	$N+4$
and so on		

For example, if in a linkage, there are 4 loops and 11 links, its degree of freedom will be 2 and the number of joints, 14. Similarly, if a linkage has 3 loops, it will require 8 links to have one degree of freedom, 9 links to have 2 degrees of freedom, 7 links to have −1 degree of freedom, etc.

Sometimes, all the above empirical relations can give incorrect results, e.g., Fig.1.16(a) has 5 links, 6 turning pairs and 2 loops. Thus, it is a structure with zero degree of freedom. However, if the links are arranged in such a way as shown in Fig. 1.16(b), a *double parallelogram linkage* with one degree of freedom is obtained. This is due to the reason that the lengths of the links or other dimensional properties are not considered in these empirical relations. So, exceptions are bound to come with equal lengths or parallel links.

Sometimes, a system may have one or more links which do not introduce any extra constraint. Such links are known as *redundant links* and should not be counted to find the degree of freedom. For example, the mechanism of Fig. 1.16(b) has 5 links, but the function of the mechanism is not affected even if any one of the links 2, 4 or 5 are removed. Thus, the effective number of links in this case is 4 with 4 turning pairs, and thus has one degree of freedom.

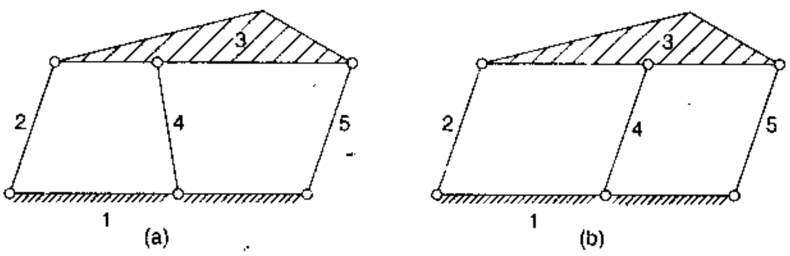

Fig. 1.16

Sometimes, one or more links of a mechanism can be moved without causing any motion to the rest of the links of the mechanism. Such a link is said to have a *redundant degree of freedom*. Thus in a mechanism, it is necessary to recognize such links prior to investigate the degree of freedom of the whole mechanism. For example, in the mechanism shown in Fig. 1.17, roller 3 can rotate about its axis without causing any movement to the rest of the mechanism. Thus, the mechanism represents a redundant degree of freedom.

In case of a mechanism possessing some redundant degree of freedom, the effective degree of freedom is given by

$$F = 3(N-1) - 2P_1 - 1P_2 - F_r$$

where F_r is the number of redundant degrees of freedom. Now, as the above mechanism has a cam pair, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 4

Number of pairs with 1 degree of freedom = 3

Number of pairs with 2 degrees of freedom = 1

$$F = 3(N-1) - 2P_1 - 1P_2 - F_r$$
$$= 3(4-1) - 2 \times 3 - 1 \times 1 - 1$$
$$= 1$$

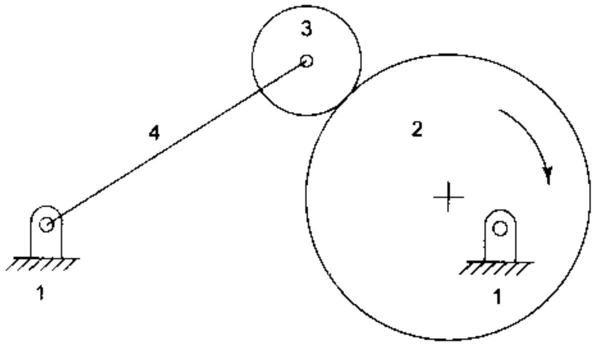

Fig. 1.17

Example 1.1 ***For the kinematic linkages shown in Fig. 1.18, calculate the following:***

- ***the number of binary links (N_b)***
- ***the number of ternary links (N_t)***
- ***the number of other (quaternary, etc.) links (N_o)***
- ***the number of total links (N)***
- ***the number of loops (L)***
- ***the number of joints or pairs (P_1)***
- ***the number of degrees of freedom (F)***

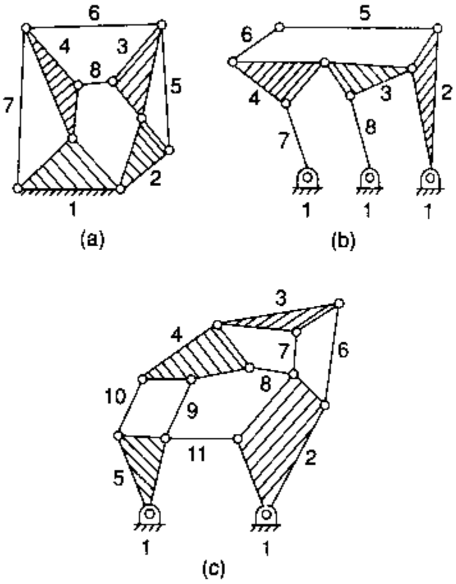

Fig. 1.18

Solution

(a) $N_b = 4$; $N_t = 4$; $N_o = 0$; $N = 8$; $L = 4$

$P_1 = 11$ by counting

or $P_1 = (N + L - 1) = 11$

$F = 3(N-1) - 2P_1$

$= 3(8-1) - 2 \times 11 = -1$

or $F = N - (2L + 1)$

$= 8 - (2 \times 4 + 1) = -1$

The linkage has negative degree of freedom and thus is a superstructure.

(b) $N_b = 4$; $N_t = 4$; $N_o = 0$; $N = 8$; $L = 3$

$P_1 = 10$ (by counting)

or $P_1 = (N + L - 1) = 10$

$F = N - (2L + 1) = 8 - (2 \times 3 + 1) = 1$

or $F = 3(N-1) - 2P_1$

$= 3(8-1) - 2 \times 10 = 1$

i.e., the linkage has a constrained motion when one of the seven moving links is driven by an external source.

(c) $N_b = 7$; $N_t = 2$; $N_o = 2$; $N = 11$

$L = 5$; $P_1 = 15$

$F = N - (2L + 1) = 11 - (2 \times 5 + 1) = 0$

Therefore, the linkage is a structure.

Example 1.2 ***State whether the linkages shown in Fig. 1.19 are mechanisms with one degree of freedom. If not, make suitable changes. The number of links should not be varied by more than 1.***

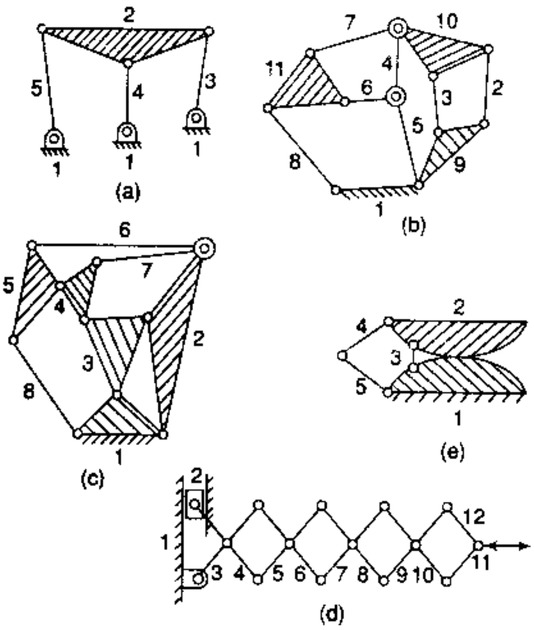

Fig. 1.19

Solution (a) The linkage has 2 loops and 5 links.

$F = N - (2L + 1) = 5 - (2 \times 2 + 1) = 0$

Thus, it is a structure. Referring Table 1.2, for a 2-loop mechanism, n should be six to have one degree of freedom. Thus, one more link should be added to the linkage to make it a mechanism of $F = 1$. One of the possible solutions has been shown in Fig. 1.20(a).

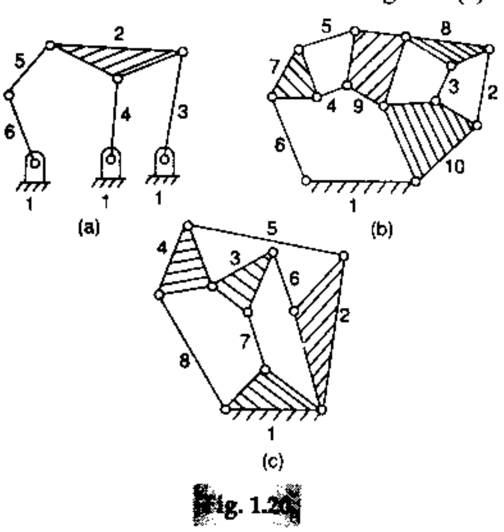

Fig. 1.20

(b) The linkage has [illegible] loops and 11 links. Referring Table 1.2, it has 2 degrees of freedom. With 4 loops and 1 degree of freedom, the number of links should be 10 and the number of joints 13. Three excess joints can be formed by

6 ternary links or
4 ternary links and 1 quaternary link or
2 ternary links, and 2 quaternary links, or
3 quaternary links, or
a combination of ternary and quaternary links with double joints.

Figure 1.20(b) shows one of the possible solutions.

(c) There are 4 loops and 8 links.

$F = N - (2L +) = 8 - (4 \times 2 + 1) = -1$

It is a superstructure. With 4 loops, the number of links must be 10 to obtain one degree of freedom. As the number of links is not to be increased by more than one, the number of loops has to be decreased. With 3 loops, 8 links and 10 joints, the required linkage can be designed. One of the many solutions is shown in Fig. 1.20(c).

(d) It has 5 loops and 12 links. Referring Table 1.2, it has 1 degree of freedom and thus is a mechanism.

(e) The mechanism has a cam pair, therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 5

Number of pairs with 1 degree of freedom = 5

Number of pairs with 2 degrees of freedom = 1

$F = 3(N - 1) - 2P_1 - P_2$

$= 3(5 - 1) - 2 \times 5 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.

Example 1.3 ***Determine the degree of freedom of the mechanisms shown in Fig. 1.21.***

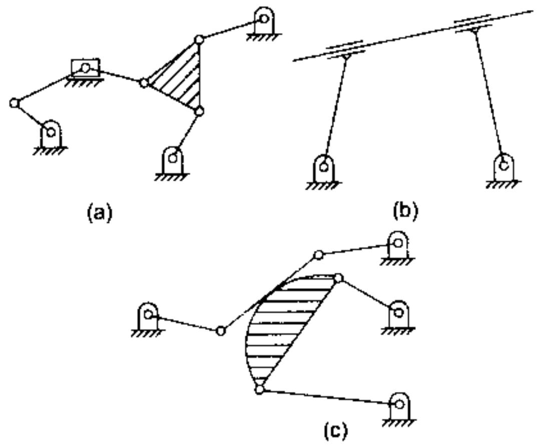

Fig. 1.21

Solution

(a) The mechanism has a sliding pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 8 (Fig. 1.22)

Number of pairs with 1 degree of freedom = 10

(At the slider, one sliding pair and two turning pairs)

$F = 3(N - 1) - 2P_1 - P_2$

$= 3(8 - 1) - 2 \times 10 - 0 = 1$

Thus, it is a mechanism with a single degree of freedom.

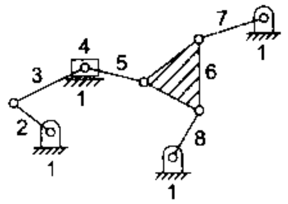

Fig. 1.22

(b) The system has a redundant degree of freedom as the rod of the mechanism can slide without causing any movement in the rest of the mechanism.

$\therefore$ effective degree of freedom

$= 3(N - 1) - 2P_1 - P_2 - F_r$

$= 3(4 - 1) - 2 \times 4 - 0 - 1 = 0$

As the effective degree of freedom is zero, it is a locked system.

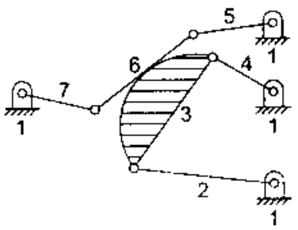

Fig. 1.23

(c) The mechanism has a cam pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 7 (Fig. 1.23)

Number of pairs with 1 degree of freedom = 8

Number of pairs with 2 degrees of freedom = 1

$F = 3(N - 1) - 2P_1 - P_2$

$= 3(7 - 1) - 2 \times 8 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.

Example 1.4 *How many unique mechanisms can be obtained from the 8-link kinematic chain shown in Fig. 1.24?*

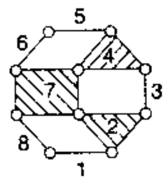

Fig. 1.24

Solution The kinematic chain has 8 links in all. A unique mechanism is obtained by fixing one of the links to the ground each time and retaining only one out of the symmetric mechanisms thus obtained.

The given kinematic chain is symmetric about links 3 or 7. Thus, identical inversions (mechanisms) are obtained if the links 2, 1, 8 or 4, 5, 6 are fixed. In addition, two more unique mechanisms can be obtained from the 8-link kinematic chain as shown in Fig. 1.25.

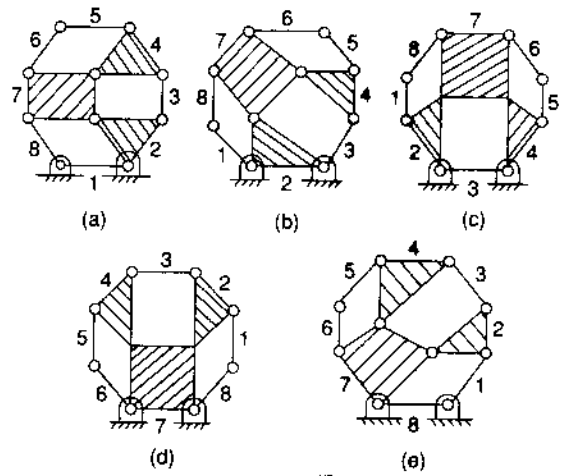

Fig. 1.25

Example 1.5 *A linkage has 11 links and 4 loops. Calculate its degree of freedom and the number of ternary and quaternary links it will have if it has only single turning pairs.*

Solution $F = N - (2L + 1) = 11 - (2 \times 4 + 1) = 2$

$P_1 = N + (L - 1) = 11 + (4 - 1) = 14$

The linkage has 3 excess joints and if all the joints are single turning pairs, the excess joints can be provided either by

- 6 ternary links or
- 4 ternary links and one quaternary link or
- 2 ternary links and two quaternary links or
- 3 quaternary links

1.12 EQUIVALENT MECHANISMS

It is possible to replace turning pairs of plane mechanisms by other types of pairs having one or two degrees of freedom, such as sliding pairs or cam pairs. This can be done according to some set rules so that the new mechanisms also have the same degrees of freedom and are kinematically similar.

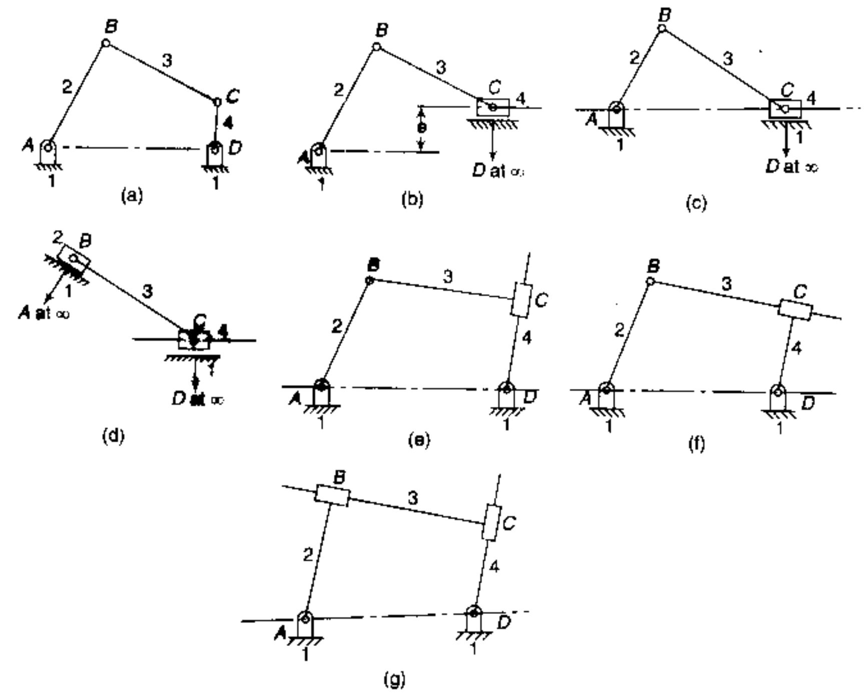

Fig. 1.26

1. Sliding Pairs in Place of Turning Pairs

Figure 1.26(a) shows a four-link mechanism. Let the length of the link 4 be increased to infinity so that *D* lies at infinity. Now, with the rotation of the link 2, *C* will have a linear motion perpendicular to the axis of the link 4. The same motion of *C* can be obtained if the link 4 is replaced by a slider, and guides are provided for its motion as shown in Fig. 1.26(b). In this case, the axis of the slider does not pass through *A* and there is an eccentricity. Figure 1.26(c) shows a slider-crank mechanism with no eccentricity. In this way, a binary link is replaced by a slider pair.

Note that the axis of the sliding pair must be in the plane of the linkage or parallel to it.

Similarly, the turning pair at *A* can also be replaced by a sliding pair by providing a slider with guides at *B* [Fig. 1.26(d)].

In case the axes of the two sliding pairs are in one line or parallel, the two sliders along with the link 3 act as one link with no relative motion among these links. Then the arrangement ceases to be a linkage. Thus, in order to replace two turning pairs in a linkage with sliding pairs, the axes of the sliding pairs must intersect.

In the same way, the turning pairs at *B* and *C* can be replaced by sliding pairs by fixing a slider to any of the two links forming the pair [Figs 1.26(e) and (f)]. Figure 1.26(g) shows both of the turning pairs at *B* and *C* replaced by sliding pairs.

2. Spring in Place of Turning Pairs

The action of a spring is to elongate or to shorten as it becomes in tension or in compression. A similar variation in length is accomplished by two binary links joined by a turning pair. In Fig. 1.27(a), the length *AB* varies as *OB* is moved away or towards point *A*. Figure 1.27(b) shows a 6-link mechanism in which links 4 and 5 have been shown replaced by a spring.

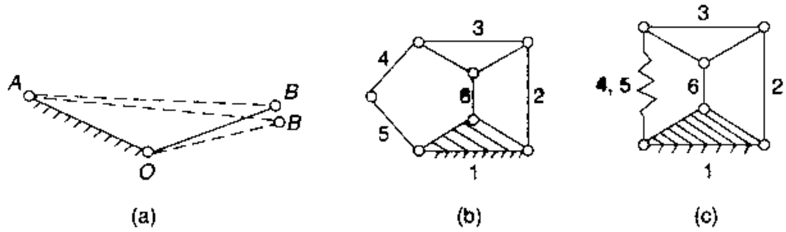

Fig. 1.27

Remember that the spring is not a rigid link but is simulating the action of two binary links joined by a turning pair. Therefore, to find the degree of freedom of such a mechanism, the spring has to be replaced by the binary links.

3. Cam Pair in Place of Turning Pair

A cam pair has two degrees of freedom. For linkages with one degree of freedom, application of Gruebler's equation yields,

$$F = 3(N - 1) - 2P_1 - 1P_2$$

or $$1 = 3N - 3 - 2P_1 - 1 \times 1$$

or $$P_1 = \frac{3N - 5}{2}$$

This shows that to have one cam pair in a mechanism with one degree of freedom, the number of links and turning pairs should be as below:

$N = 3, \quad P_1 = 2$
$N = 5, \quad P_1 = 5$
$N = 7, \quad P_1 = 8$
$N = 9, \quad P_1 = 11$ and so on.

A comparison of this with linkages having turning pairs only (Table 1.2) indicates that a cam pair can be replaced by one binary link with two turning pairs at each end.

Figure 1.28(a) shows link *CD* (of a four-link mechanism) with two turning pairs at its ends replaced by a cam pair. The centres of curvatures at the point of contact *X* of the two cams lie at *D* and *C*. Figures 1.28(b) and (c) show the link *BC* with turning pairs at *B* and *C* replaced by a cam pair. The centres of curvature at the point of contact *X* lie at *B* and *C* respectively. Figure 1.28(d) shows equivalent mechanism for a disc cam with reciprocating curved-face follower. The centres of curvature of the cam and the follower at the instant lie at *A* and *B* respectively.

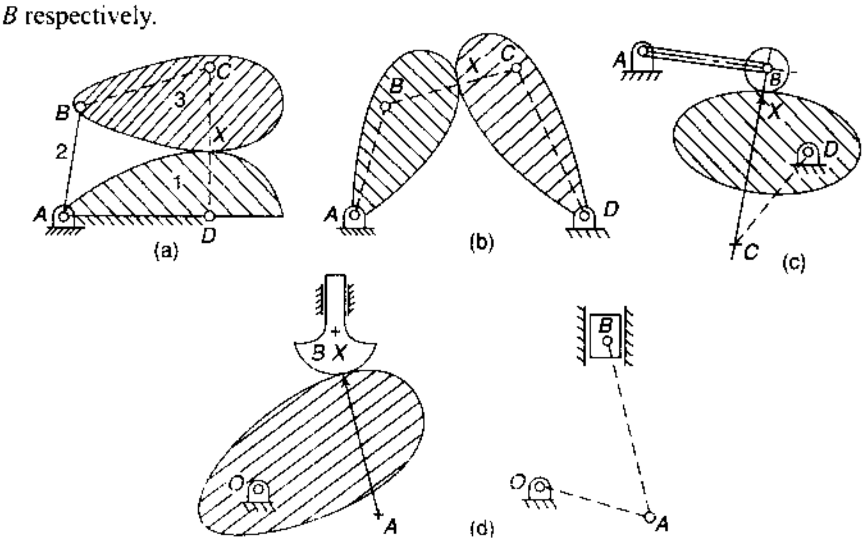

Fig. 1.28

Example 1.6 *Sketch a few slider-crank mechanisms derived from Stephenson's and Watt's six-bar chains.*

Solution Figure 1.29(a) shows a Stephenson's chain in which the ternary links are not directly connected. Thus, any of the binary links 3 or 6 can be replaced by a slider to obtain a slider-crank mechanism as shown in Fig. 1.29(b) and (c).

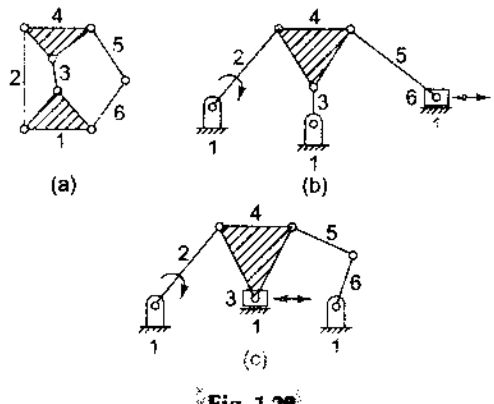

Fig. 1.29

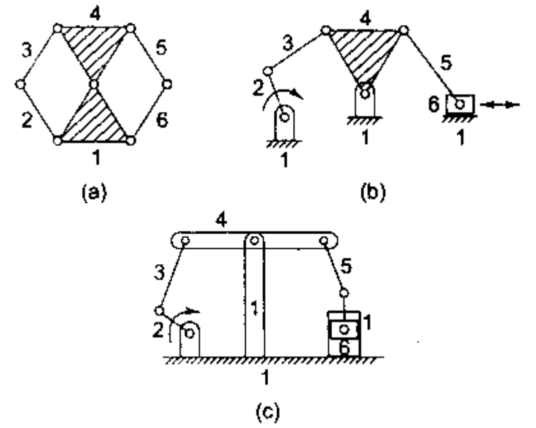

Fig. 1.30

Figure 1.30(a) shows a Watt's chain in which the ternary links are directly connected. Thus, any of the binary links 2 or 6 can be replaced by a slider to obtain a slider-crank mechanism. Figure 1.30 (b) and (c) show two variations of the slider obtained by replacing the binary link 6. The slider-crank mechanism of Fig. 1.30(c) is known as *beam engine*.

Example 1.7 ***Sketch the equivalent kinematic chains with turning pairs for the chains shown in Fig. 1.31.***

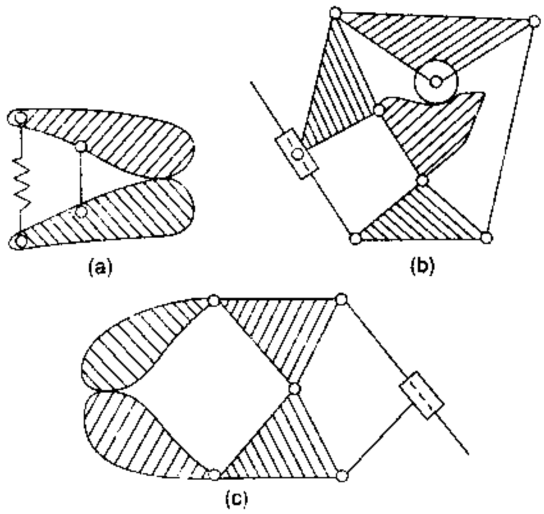

Fig. 1.31

Solution

(a) A spring is equivalent to two binary links connected by a turning pair. A cam pair is equivalent of one binary link with turning pairs at each end. The equivalent chain with turning pairs is shown in Fig. 1.32(a).

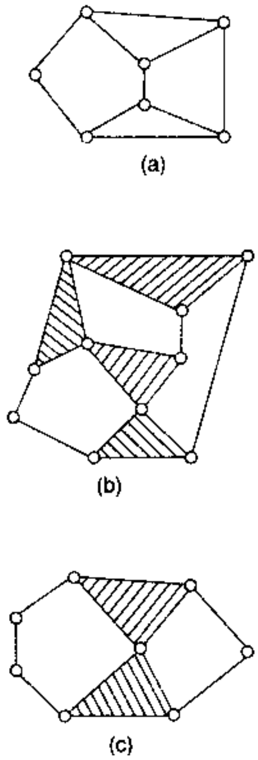

Fig. 1.32

(b) A slider pair can be replaced by one link with a turning pair at the other end. A cam pair with a roller follower can be replaced by a binary link with turning pairs at each end similar to the case of a curved-face follower of Fig. 1.28(d). the equivalent chain is shown in Fig. 1.32(b).

(c) The equivalent chain has been shown in Fig. 1.32(c).

1.13 THE FOUR-BAR CHAIN

A four-bar chain is the most fundamental of the plane kinematic chains. It is a much preferred mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints. When one of the links is fixed, it is known as a *linkage* or *mechanism*. A link that makes complete revolution is called the *crank*, the link opposite to the fixed link is called the *coupler*, and the fourth link is called a *lever* or *rocker* if it oscillates or another crank, if it rotates.

Note that it is impossible to have a four-bar linkage if the length of one of the links is greater than the sum of the other three. This has been shown in Fig. 1.33 in which the length of link d is more than the sum of lengths of a, b and c, and therefore, this linkage cannot exist.

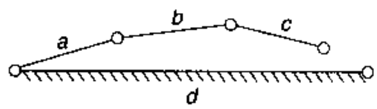

Fig. 1.33

Consider a four-link mechanism shown in Fig. 1.34(a) in which the length a of the link AB is more than d, the length of the fixed link AD. The linkage has been shown in various positions. It can be observed from these configurations that if the link a is to rotate through a full revolution, i.e., if it is to be a crank, then the following conditions must be met:

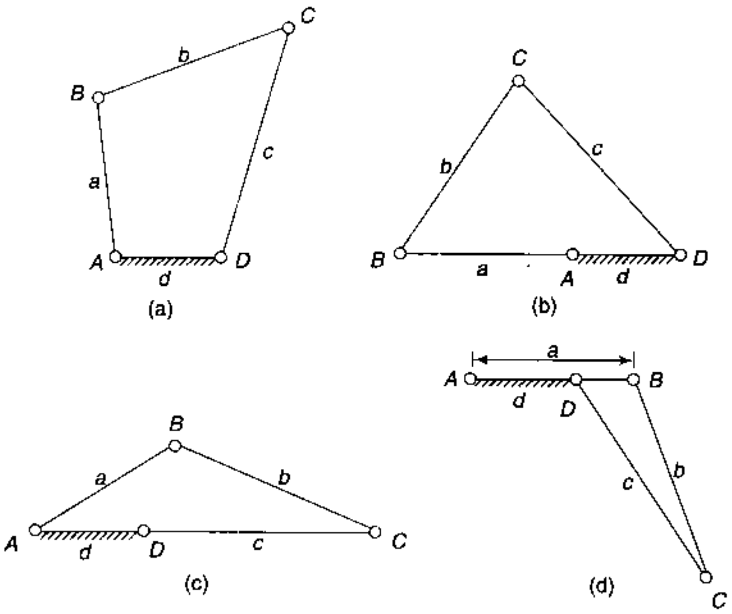

Fig. 1.34

From Fig. 1.34(b), $\quad d + a < b + c$ (i)

From Fig. 1.34(c), $\quad d + c < a + b$ (ii)

From Fig. 1.34(d), $\quad b < c + (a - d) \quad$ or $\quad d + b < c + a$ (iii)

Adding (i) and (ii), $\quad 2d + a + c < 2b + a + c$

or $\quad d < b$

Similarly, adding (ii) and (iii), and (iii) and (i) we get

$$d < a$$

and $\quad d < c$

Thus, d is less than a, b and c, i.e., it is the shortest link if a is to rotate a full circle or act as a crank. The above inequalities also suggest that out of a, b and c, whichever is the longest, the sum of that with d, the shortest link will be less than the sum of the remaining two links. Thus, the necessary conditions for the link a to be a crank is

- the shortest link is fixed, and
- the sum of the shortest and the longest links is less than the sum of the other two links.

In a similar way, it can be shown that if the link c is to rotate through a full circle, i.e., if it is to be a crank then the conditions to be realised are the same as above. Also, it can be shown that if both the links a and c rotate through full circles, the link b also makes one complete revolution relative to the fixed link d.

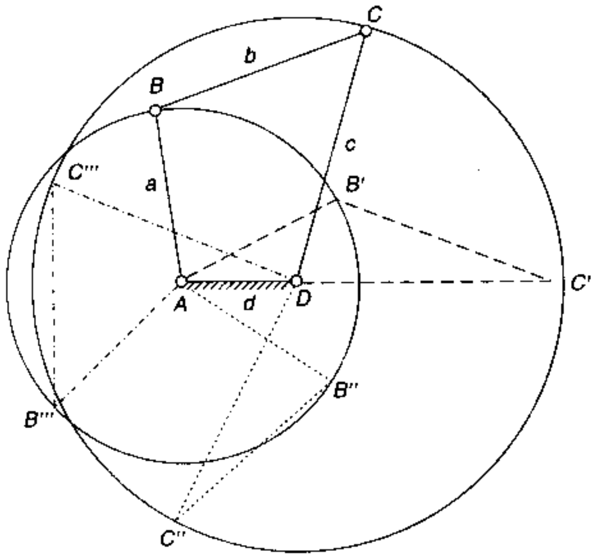

Fig. 1.35

The mechanism thus obtained is known as *crank-crank* or *double-crank* or *drag-crank mechanism* or *rotary-rotary converter*. Figure 1.35 shows all the three links a, b and c rotating through one complete revolution.

In the above consideration, the rotation of the links is observed relative to the fixed link d. Now, consider the movement of b relative to either a or c. The complete rotation of b relative to a is possible if the angle $\angle ABC$ can be more than 180° and relative to c if the angle $\angle DCB$ more than 180°. From the positions of the links in Fig. 1.35(b) and (c), it is clear that these angles cannot become more than 180° for the above stated conditions.

Now, as the relative motion between two adjacent links remains the same irrespective of which link is fixed to the frame, different mechanisms (known as *inversions*) obtained by fixing different links of this kind of chain will be as follows:

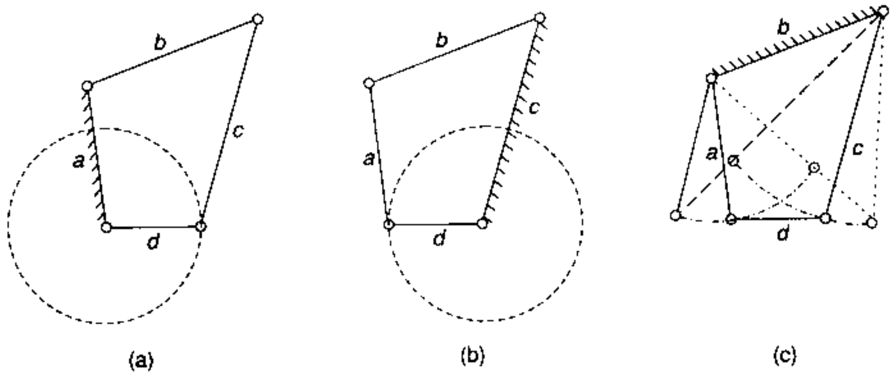

Fig. 1.36

1. If any of the adjacent links of link *d*, i.e., *a* or *c* is fixed, *d* can have a full revolution (crank) and the link opposite to it oscillates (rocks). In Fig. 1.36(a), *a* is fixed, *d* is the crank and *b* oscillates whereas in Fig. 1.36(b), *c* is fixed, *d* is the crank and *b* oscillates. The mechanism is known as *crank-rocker* or *crank-lever mechanism* or *rotary-oscillating converter.*
2. If the link opposite to the shortest link, i.e., link *b* is fixed and the shortest link *d* is made a coupler, the other two links *a* and *c* would oscillate [Fig. 1.36(c)]. The mechanism is known as a *rocker-rocker* or *double-rocker* or *double-lever mechanism* or *oscillating-oscillating converter.*

A linkage in which the sum of the lengths of the longest and the shortest links is less than the sum of the lengths of the other two links, is known as a *class-I,* four-bar linkage.

When the sum of the lengths of the largest and the shorted links is more than the sum of the lengths of the other two links, the linkage is known as a *class-II,* four-bar linkage. In such a linkage, fixing of any of the links always results in a *rocker-rocker* mechanism. In other words, the mechanism and its inversions give the same type of motion (of a *double-rocker* mechanism).

The above observations are summarised in *Grashof's law* which states *that a four-bar mechanism has at least one revolving link if the sum of the lengths of the largest and the shortest links is less than the sum of lengths of the other two links.*

Further, if the *shortest link is fixed,* the chain will act as a double-crank mechanism in which links adjacent to the fixed link will have complete revolutions. If the *link opposite to the shortest link is fixed,* the chain will act as double-rocker mechanism in which links adjacent to the fixed link will oscillate. If the *link adjacent to the shortest link is fixed,* the chain will act as crank-rocker mechanism in which the shortest link will revolve and the link adjacent to the fixed link will oscillate.

If the sum of the lengths of the largest and the shorted links is equal to the sum of the lengths of the other two links, i.e., when equalities exist, the four inversions, in general, result in mechanisms similar to those as given by Grashof's law, except that sometimes the links may become collinear and may have to be guided in the proper direction. Usually, the purpose is served by the inertia of the links. A few special cases may arise when equalities exist. For example, *parallel-crank four-bar linkage* and *deltoid* linkage.

Parallel-Crank Four-Bar Linkage If in a four-bar linkage, two opposite links are parallel and equal in length, then any of the links can be made fixed. The two links adjacent to the fixed link will always act as two cranks. The four links form a parallelogram in all the positions of the cranks, provided the cranks rotate in the same sense as shown in Fig. 1.37.

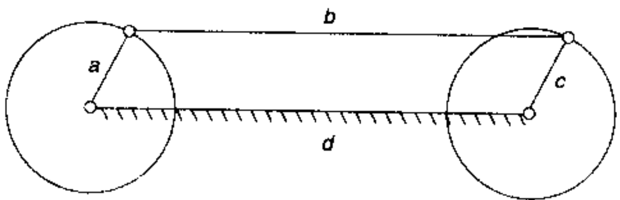

Fig. 1.37

The use of such a mechanism is made in the coupled wheels of a locomotive in which the rotary motion of one wheel is transmitted to the other wheel. For kinematic analysis, link *d* is treated as fixed and the relative motions of the other links are found. However, in fact, *d* has a translatory motion parallel to the rails.

Deltoid Linkage In a deltoid linkage (Fig. 1.38), the equal links are adjacent to each other. When any of the shorter links is fixed, a double-crank mechanism is obtained in which one revolution of the longer link causes two revolutions of the other shorter link. As shown in Fig. 1.38 (a), when the link *c* rotates through half a revolution and assumes the position *DC'*, the link *a* has completed a full revolution.

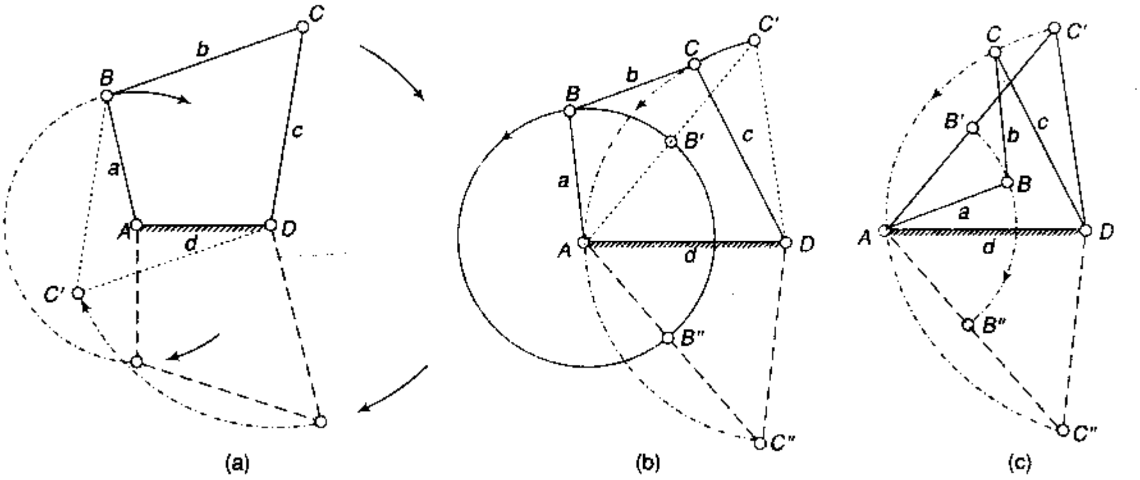

Fig. 1.38

When any of the longer links is fixed, two crank-rocker mechanisms are obtained [Fig. 1.38(b) and (c)]

Example 1.8 ***Find all the inversion of the chain given in Fig. 1.39.***

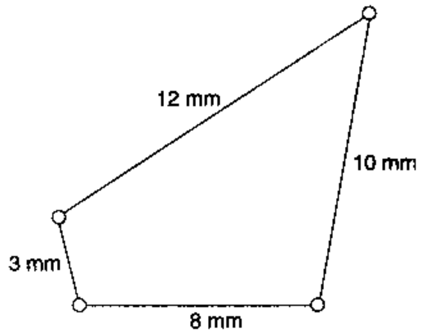

Fig. 1.39

Solution

(a) Length of the longest link = 12 mm
Length of the shortest link = 3 mm
Length of other links = 10 mm and 8 mm
Since 12 + 3 < 10 + 8, it belongs to the class-I mechanism and according to Grashoff's law, three distinct inversions are possible.

Shortest link fixed, i.e., when the link with 3-mm length is fixed, the chain will act as double-crank mechanism in which links with lengths of 12 mm and 8 mm will have complete revolutions.

Link opposite to the shortest link fixed, i.e., when the link with 10-mm length is fixed, the chain will act as double-rocker mechanism in which links with lengths of 12 mm and 8 mm will oscillate.

Link adjacent to the shortest link fixed, i.e., when any of the links adjacent to the shortest link, i.e., link with a length of 12-mm or 8 mm is fixed, the chain will act as crank-rocker mechanism in which the shortest link of 3-mm length will revolve and the link with 10-mm length will oscillate.

Example 1.9 ***Figure 1.40 shows some four-link mechanisms in which the figures indicate the dimensions in standard units of length. Indicate the type of each mechanism whether crank-rocker or double-crank or double-rocker.***

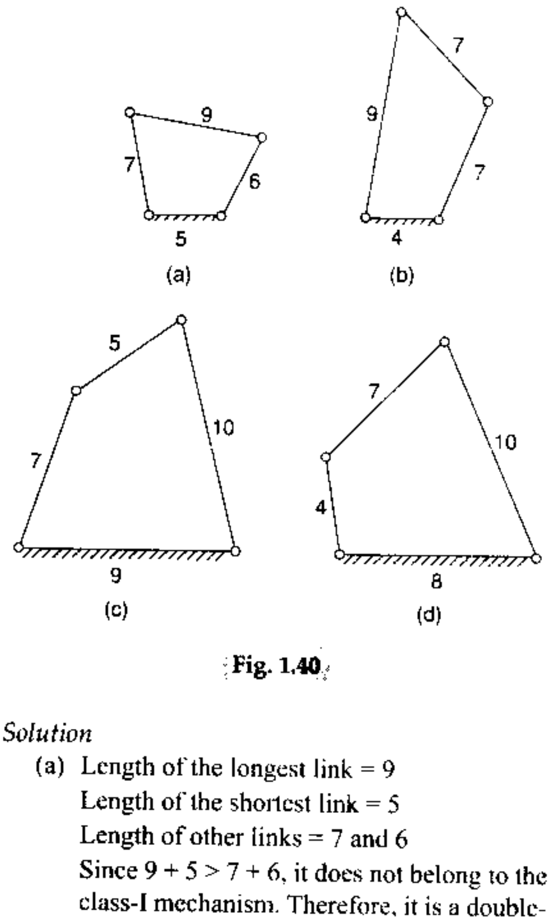

Fig. 1.40

Solution

(a) Length of the longest link = 9
Length of the shortest link = 5
Length of other links = 7 and 6
Since 9 + 5 > 7 + 6, it does not belong to the class-I mechanism. Therefore, it is a double-rocker mechanism.

(b) Length of the longest link = 9
Length of the shortest link = 4
Length of other links = 7 and 7
Since 9 + 4 < 7 + 7, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism.

(c) Length of the longest link = 10
Length of the shortest link = 5
Length of other links = 9 and 7
Since 10 + 5 < 9 + 7, it belongs to the class-I mechanism. In this case as the link opposite to the shortest link is fixed, it is a double-rocker mechanism.

(d) Length of the longest link = 10
Length of the shortest link = 4
Length of other links = 8 and 7
Since 10 + 4 < 8 + 7, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Example 1.10 *Figure 1.41 shows a plane mechanism in which the figures indicate the dimensions in standard units of length. The slider C is the driver. Will the link AG revolve or oscillate?*

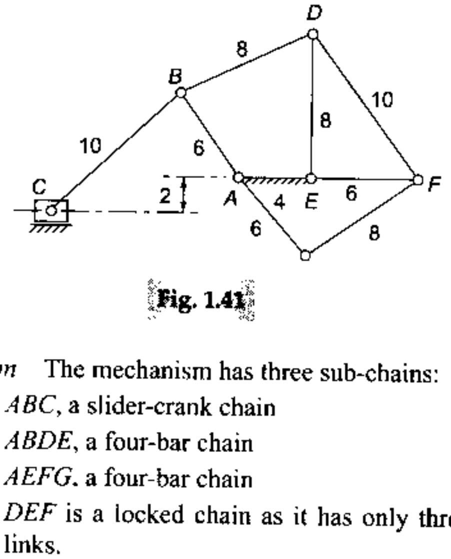

Fig. 1.41

Solution The mechanism has three sub-chains:

(i) *ABC*, a slider-crank chain
(ii) *ABDE*, a four-bar chain
(iii) *AEFG*, a four-bar chain
DEF is a locked chain as it has only three links.

- As the length *BC* is more than the length *AB* plus the offset of 2 units, *AB* acts as a crank and can revolve about *A*.
- In the chain *ABDE*,
Length of the longest link = 8
Length of the shortest link = 4
Length of other links = 8 and 6

Since 8 + 4 < 8 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism and thus *AB* and *ED* can revolve fully.

- In the chain *AEFG*,
Length of the longest link = 8
Length of the shortest link = 4
Length of other links = 6 and 6

Since 8 + 4 = 6 + 6, it belongs to the class-I mechanism. As the shortest link is fixed, it is a double-crank mechanism and thus *EF* and *AG* can revolve fully.

As *DEF* is a locked chain with three links, the link *EF* revolves with the revolving of *ED*. With the revolving of *ED*, *AG* also revolves.

1.14 MECHANICAL ADVANTAGE

The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant. Thus for the linkage of Fig. 1.42, if friction and inertia forces are ignored and the input torque T_2 is applied to the link 2 to drive the output link 4 with a resisting torque T_4 then

Power input = Power output

$$T_2\,\omega_2 = T_4\,\omega_4$$

or $$\text{MA} = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

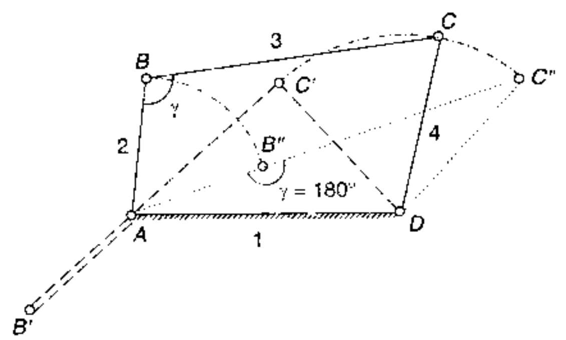

Fig. 1.42

Thus, it is the reciprocal of the velocity ratio. In case of crank-rocker mechanisms, the velocity ω_4 of the output link *DC* (rocker) becomes zero at the extreme positions ($AB'C''D$ and $AB''C''D$), i.e., when the input link *AB* is in line with the coupler *BC* and the angle γ between them is either zero or 180°, it makes the mechanical advantage to be infinite at such positions. Only a small input torque can overcome a large output torque load. The extreme positions of the linkage are known as *toggle* positions.

1.15 TRANSMISSION ANGLE

The angle μ between the output link and the coupler is known as *transmission angle*. In Fig. 1.43, if the link *AB* is the input link, the force applied to the output link *DC* is transmitted through the coupler *BC*. For a particular value of force in the coupler rod, the torque transmitted to the output link (about the point *D*) is maximum when the transmission angle μ is 90°. If links *BC* and *DC* become coincident, the *transmission angle* is zero and the mechanism would lock or jam. If μ deviates significantly from 90°, the torque on the output link decreases. Sometimes, it may not be sufficient to overcome the friction in the system and the mechanism may be locked or jammed. Hence μ is usually kept more than 45°. The best mechanisms, therefore, have a transmission angle that does not deviate much from 90°.

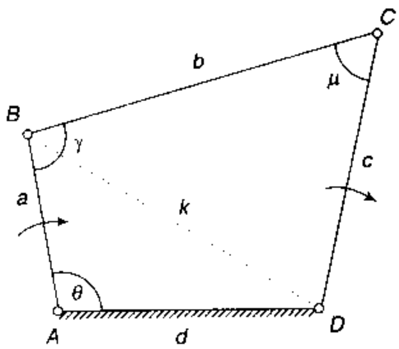

Fig. 1.43

Applying cosine law to triangles *ABD* and *BCD* (Fig. 1.43),

$$a^2 + d^2 - 2ad\cos\theta = k^2 \quad \text{(i)}$$

and $$b^2 + c^2 - 2bc\cos\mu = k^2 \quad \text{(ii)}$$

From (i) and (ii),

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

or $$a^2 + d^2 - b^2 - c^2 - 2ad\cos\theta + 2bc\cos\mu = 0$$

The maximum or minimum values of the transmission angle can be found by putting $d\mu/d\theta$ equal to zero.

Differentiating the above equation with respect to θ,

$$2\,ad \sin\theta - 2bc \sin\mu . \frac{d\mu}{d\theta} = 0$$

or $$\frac{d\mu}{d\theta} = \frac{ad \sin\theta}{bc \sin\mu}$$

Thus, if $d\mu/d\theta$ is to be zero, the term $ad \sin\theta$ has to be zero which means θ is either 0° or 180°. It can be seen that μ is maximum when θ is 180° and minimum when θ is 0°. However, this would be applicable to the mechanisms in which the link a is able to assume these angles, i.e., in double-crank or crank-rocker mechanisms. Figures 1.44(a) and (b) show a crank-rocker mechanism indicating the positions of the maximum and the minimum transmission angles. Figures 1.45(a) and (b) show the maximum and the minimum transmission angles for a double-rocker mechanism.

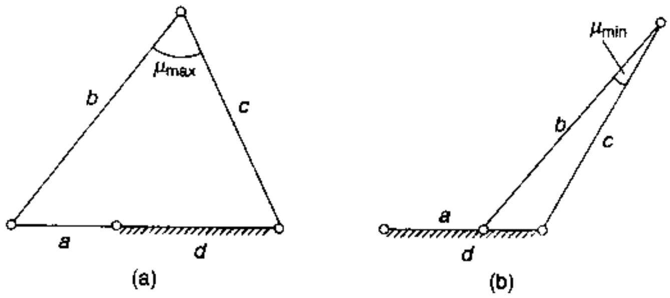

Fig. 1.44

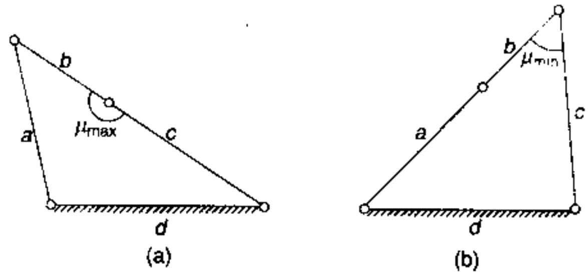

Fig. 1.45

Example 1.11 ***Find the maximum and minimum transmission angles for the mechanisms shown in Fig. 1.46. The figures indicate the dimensions in standard units of length.***

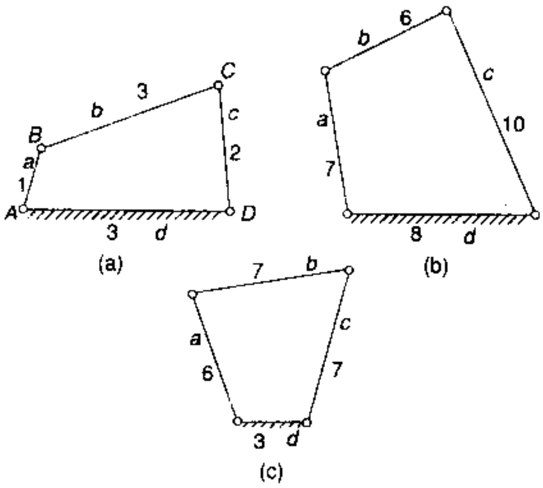

Fig. 1.46

Solution

(a) In this mechanism,

Length of the longest link = 3

Length of the shortest link = 1

Length of other links = 3 and 2

Since 3 + 1 < 3 + 2, it belongs to the class I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

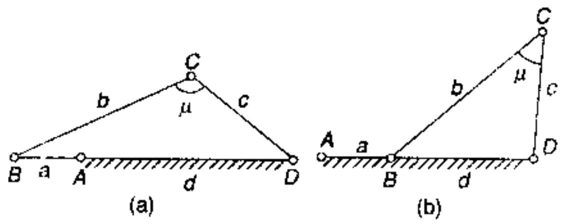

Fig. 1.47

Maximum transmission angle is when θ is 180° [Fig. 1.47(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc \cos\mu$

$(1 + 3)^2 = 3^2 + 2^2 - 2 \times 3 \times 2 \cos\mu$

$16 = 9 + 4 - 12 \cos\mu$

$$\cos \mu = -\frac{3}{12} = -0.25$$

$\mu = 104.5°$

Minimum transmission angle is when θ is 0° [Fig. 1.47(b)],

Thus $(d - a)^2 = b^2 + c^2 - 2bc \cos \mu$

$(3 - 1)^2 = 3^2 + 2^2 - 2 \times 3 \times 2 \cos \mu$

$4 = 9 + 4 - 12 \cos \mu$

$$\cos \mu = \frac{3}{4} = 0.75$$

$\mu = 41.4°$

(b) In this mechanism,

Length of the longest link = 10

Length of the shortest link = 6

Length of other links = 8 and 7

Since 10 + 6 > 8 + 7, it belongs to the class-II mechanism and thus is a double-rocker mechanism.

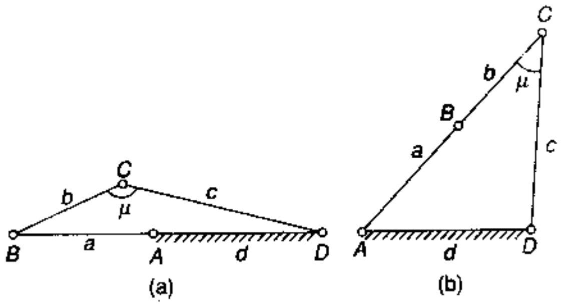

Fig. 1.48

Maximum transmission angle is when θ is 180° [Fig. 1.48(a)],

Thus, $(a + d)^2 = b^2 + c^2 - 2bc \cos \mu$

$(7 + 8)^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos \mu$

$225 = 36 + 100 - 120 \cos \mu$

$$\cos \mu = -\frac{89}{120} = -0.742$$

$\mu = 137.9°$

Minimum transmission angle is when the angle at B is 180° [Fig. 1.48(b)],

Thus, $d^2 = (a + b)^2 + c^2 - 2(a + b)c \cos \mu$

$8^2 = (7 + 6)^2 + 10^2 - 2(7 + 6) \times 10 \times \cos \mu$

$64 = 169 + 100 - 260 \cos \mu$

$$\cos \mu = \frac{205}{260} = 0.788$$

$\mu = 38°$

(c) In this mechanism,

Length of the longest link = 7

Length of the shortest link = 3

Length of other links = 6 and 6

Since 7 + 3 < 6 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank or drag-link mechanism.

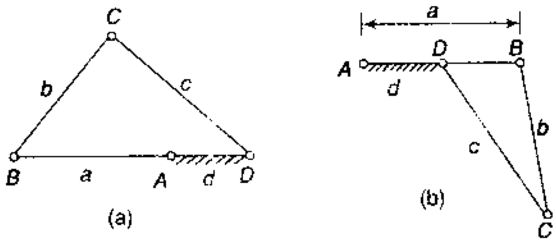

Fig. 1.49

Maximum transmission angle is when θ is 180° [Fig. 1.49(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc \cos \mu$

$(6 + 3)^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \cos \mu$

$81 = 36 + 49 - 84 \cos \mu$

$$\cos \mu = \frac{4}{84} = 0.476$$

$\mu = 87.27°$

Minimum transmission angle is when θ is 0° [Fig. 1.49(b)],

Thus $(a - d)^2 = b^2 + c^2 - 2bc \cos \mu$

$(6 - 3)^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \cos \mu$

$9 = 36 + 49 - 84 \cos \mu$

$$\cos \mu = \frac{76}{84} = 0.9048$$

$\mu = 25.2°$

Example 1.12 *A crank-rocker mechanism has a 70-mm fixed link, a 20-mm crank, a 50-mm coupler, and a 70-mm rocker. Draw the mechanism and determine the maximum and minimum values of the transmission angle. Locate the two toggle positions and find the corresponding crank angles and the transmission angles.*

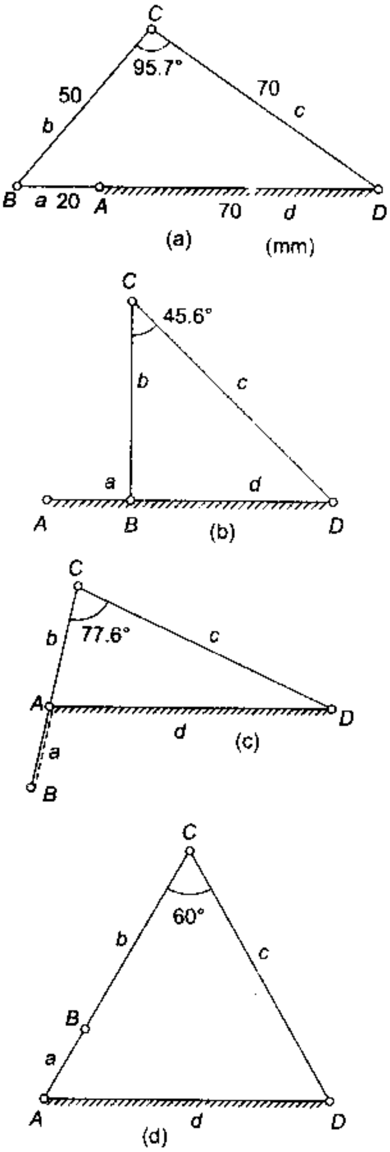

Fig. 1.50

Solution In this mechanism,

Length of the longest link = 70 mm

Length of the shortest link = 20 mm

Length of other links = 70 and 50 mm

Since 70 + 20 < 70 + 50, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Maximum transmission angle is when θ is 180° [Fig. 1.50(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc \cos \mu$

$$(20 + 70)^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos \mu$$

$$8100 = 2500 + 4900 - 7000 \cos \mu$$

$$\cos \mu = -0.1$$

$$\mu = 95.7°$$

Minimum transmission angle is when θ is 0° [Fig. 1.50(b)],

Thus $(70 - 20)^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos \mu$

$$2500 = 2500 + 4900 - 7000 \cos \mu$$

$$\cos \mu = 0.7$$

$$\mu = 45.6°$$

The two toggle positions are shown in Figs 1.50(c) and (d).

Transmission angle for first position,

$$d^2 = (b - a)^2 + c^2 - 2(b - a)c \cos \mu$$

$$70^2 = 30^2 + 70^2 - 2 \times 30 \times 70 \cos \mu$$

$$4900 = 900 + 4900 - 4200 \cos \mu$$

$$\cos \mu = 0.214$$

$$\mu = 77.6°$$

As c and d are of equal length [Fig. 1.50(c)], it is an isosceles triangle and thus input angle $\theta = (77.6° + 180°) = 257.6°$

Transmission angle for second position Fig. 1.50(d),

$$d^2 = (b + a)^2 + c^2 - 2(b + a)c \cos \mu$$

$$70^2 = 70^2 + 70^2 - 2 \times 70 \times 70 \cos \mu$$

$$4900 = 4900 + 4900 - 9800 \cos \mu$$

$$\cos \mu = 0.5$$

$$\mu = 60°$$

(or as all the sides of the triangle of Fig. 1.50(d) are of equal length, it is an equilateral triangle and thus transmission angle is equal to 60°)

And the input angle, $\theta = 60°$

- The above results can also be obtained graphically by drawing the figures to scale and measuring the angles.

1.16 THE SLIDER-CRANK CHAIN

When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a *single slider-crank chain* or simply a *slider-crank chain*. It is also possible to replace two sliding pairs of a four-bar chain to get a *double slider-crank chain* (Sec. 1.17). Further, in a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced. The distance e between the fixed pivot O and the straight line path of the slider is called the *offset* and the chain so formed an *offset slider-crank chain*.

Different mechanisms obtained by fixing different links of a kinematic chain are known as its *inversions*. A slider-crank chain has the following inversions:

First Inversion

This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and the slider respectively [Fig. 1.51(a)].

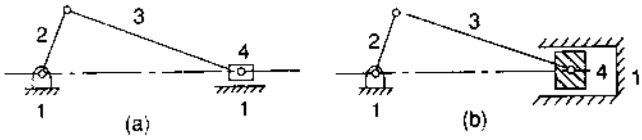

Fig. 1.51

Applications

1. Reciprocating engine
2. Reciprocating compressor

As shown in Fig. 1.51(b), if it is a reciprocating engine, 4 (piston) is the driver and if it is a compressor, 2 (crank) is the driver.

Second Inversion

Fixing of the link 2 of a slider-crank chain results in the second inversion.

The slider-crank mechanism of Fig. 1.51(a) can also be drawn as shown in Fig. 11.52(a). Further, when its link 2 is fixed instead of the link 1, the link 3 along with the slider at its end B becomes a crank. This makes the link 1 to rotate about O along with the slider which also reciprocates on it [Fig. 1.52(b)].

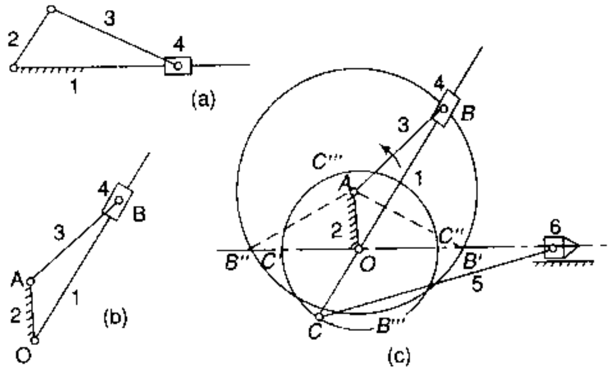

Fig. 1.52

Applications

1. Whitworth quick-return mechanism
2. Rotary engine

Whitworth Quick-Return Mechanism It is a mechanism used in workshops to cut metals. The forward stroke takes a little longer and cuts the metal whereas the return stroke is idle and takes a shorter period.

Slider 4 rotates in a circle about A and slides on the link 1 [Fig. 1.52(c)]. C is a point on the link 1 extended backwards where the link 5 is pivoted. The other end of the link 5 is pivoted to the tool, the forward stroke of which cuts the metal. The axis of motion of the slider 6 (tool) passes through O and is perpendicular to OA, the fixed link. The crank 3 rotates in the counter-clockwise direction.

Initially, let the slider 4 be at B' so that C be at C'. Cutting tool 6 will be in the extreme left position. With the movement of the crank, the slider traverses the path $B'BB''$ whereas the point C moves through $C'CC''$. Cutting tool 6 will have the forward stroke. Finally, the slider B assumes the position B'' and the cutting tool 6 is in the extreme right position. The time taken for the forward stroke of the slider 6 is proportional to the obtuse angle $B''AB'$ at A.

Similarly, the slider 4 completes the rest of the circle through the path $B''B'''B'$ and C passes through $C'C'''C'$. There is backward stroke of the tool 6. The time taken in this is proportional to the acute angle $B''AB'$ at A.

Let

θ = obtuse angle $B'AB''$ at A

β = acute angle $B'AB''$ at A

Then,

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$


A nine-cylinder rotary engine

Rotary Engine Referring Fig. 1.52(b), it can be observed that with the rotation of the link 3, the link 1 rotates about O and the slider 4 reciprocates on it. This also implies that if the slider is made to reciprocate on the link 1, the crank 3 will rotate about A and the link 1 about O.

In a rotary engine, the slider is replaced by a piston and the link 1 by a cylinder pivoted at O. Moreover, instead of one cylinder, seven or nine cylinders symmetrically placed at regular intervals in the same plane or in parallel planes, are used. All the cylinders rotate about the same fixed centre and form a balanced system. The fixed link 2 is also common to all cylinders (Fig. 1.53).

Thus, in a rotary engine, the crank 2 is fixed and the body 1 rotates whereas in a reciprocating engine (1st inversion), the body 1 is fixed and the crank 2 rotates.

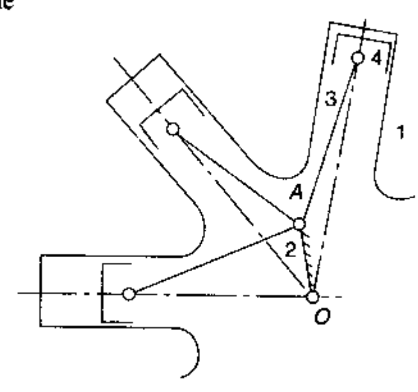

Fig. 1.53

Third Inversion

By fixing the link 3 of the slider-crank mechanism, the third inversion is obtained [Fig. 1.54(a)]. Now the link 2 again acts as a crank and the link 4 oscillates.

Applications

1. Oscillating cylinder engine
2. Crank and slotted-lever mechanism

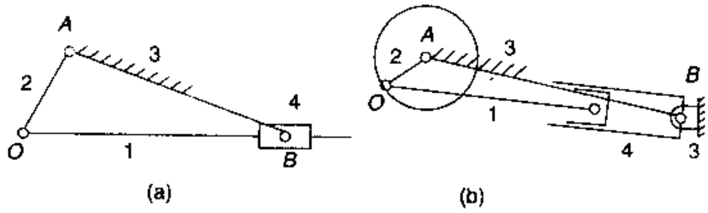

Fig. 1.54

Oscillating Cylinder Engine As shown in Fig. 1.54(b), the link 4 is made in the form of a cylinder and a piston is fixed to the end of the link 1. The piston reciprocates inside the cylinder pivoted to the fixed link 3. The arrangement is known as oscillating cylinder engine, in which as the piston reciprocates in the oscillating cylinder, the crank rotates.

Crank and Slotted-Lever Mechanism If the cylinder of an oscillating cylinder engine is made in the form of a guide and the piston in the form of a slider, the arrangement as shown in Fig. 1.55(a) is obtained. As the crank rotates about A, the guide 4 oscillates about B. At a point C on the guide, the link 5 is pivoted, the other end of which is connected to the cutting tool through a pivoted joint.

Figure 1.55(b) shows the extreme positions of the oscillating guide 4. The time of the forward stroke is proportional to the angle θ whereas for the return stroke, it is proportional to angle β, provided the crank rotates clockwise.

Comparing a crank and slotted-lever quick-return mechanism with a Whitworth quick-return mechanism, the following observations are made:

1. Crank 3 of the Whitworth mechanism is longer than its fixed link 2 whereas the crank 2 of the slotted-lever mechanism is shorter than its fixed link 3.
2. Coupler link 1 of the Whitworth mechanism makes complete rotations about its pivoted joint O with the fixed link. However, the coupler link 4 of the slotted-lever mechanism oscillates about its pivot B.
3. The coupler link holding the tool can be pivoted to the main coupler link at any convenient point C in both cases. However, for the same displacement of the tool, it is more convenient if the point C is taken on the extension of the main coupler link (towards the pivot with the fixed link) in case of the Whitworth mechanism and beyond the extreme position of the slider in the slotted-lever mechanism.

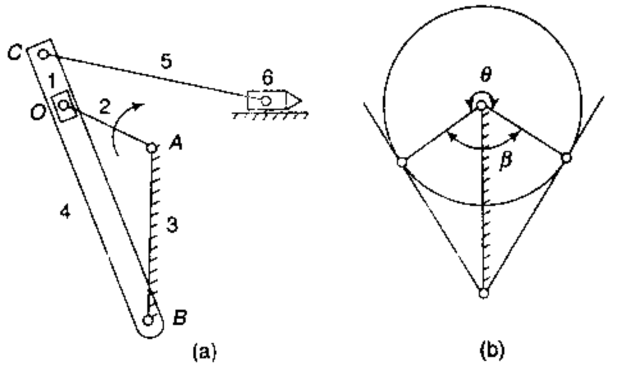

Fig. 1.55

A shaping machine. Shaping machines are fitted with quick-return mechanisms.

Fourth Inversion

If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained [Fig. 1.56(a)]. Link 3 can oscillate about the fixed pivot B on the link 4. This makes the end A of the link 2 to oscillate about B and the end O to reciprocate along the axis of the fixed link 4.

Application Hand-pump

Figure 1.56(b) shows a hand-pump. Link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

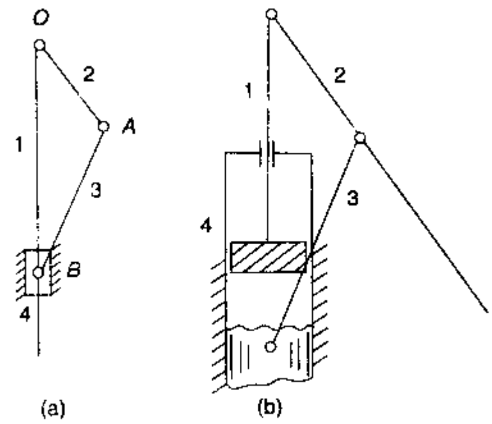

Fig. 1.56

Example 1.13 *The length of the fixed link of a crank and slotted-lever mechanism is 250 mm and that of the crank is 100 mm. Determine the*

(i) inclination of the slotted lever with the vertical in the extreme position,

(ii) ratio of the time of cutting stroke to the time of return stroke, and

(iii) length of the stroke, if the length of the slotted lever is 450 mm and the line of stroke passes through the extreme positions of the free end of the lever.

Solution Refer Fig. 1.57.

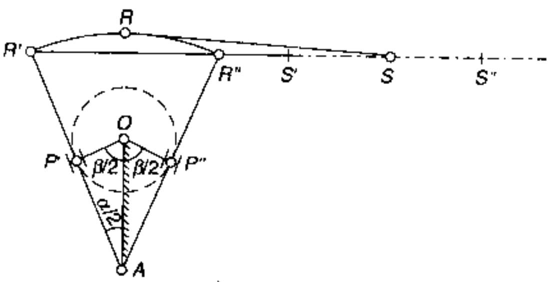

Fig. 1.57

$OA = 250$ mm $\quad OP' = OP'' = 100$ mm

$AR' = AR'' = AR = 450$ mm

$$\cos\frac{\beta}{2} = \frac{OP'}{OA} = \frac{100}{250} = 0.4$$

or $\quad \frac{\beta}{2} = 66.4° \quad$ or $\quad \beta = 132.8°$

(i) Angle of the slotted lever with the vertical

$\alpha/2 = 90° - 66.4° = 23.6°$

(ii) $\dfrac{\text{Time of cutting stroke}}{\text{Time of return stroke}}$

$$= \frac{360° - \beta}{\beta} = \frac{360° - 132.8°}{132.8°} = 1.71$$

(iii) Length of stroke $= S'S'' = R'R''$

$= 2\,AR'.\sin(\alpha/2)$

$= 2 \times 450 \sin 23.6°$

$= 360.3$ mm

1.17 DOUBLE SLIDER-CRANK CHAIN

A four-bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as a double-slider-crank chain [Fig. 1.58(a)]. The following are its inversions.

First Inversion

This inversion is obtained when the link 1 is fixed and the two adjacent pairs 23 and 34 are turning pairs and the other two pairs 12 and 41 sliding pairs.

Application Elliptical trammel

Elliptical Trammel Figure 1.58(b) shows an elliptical trammel in which the fixed link 1 is in the form of guides for sliders 2 and 4. With the movement of the sliders, any point C on the link 3, except the midpoint of AB will trace an ellipse on a fixed plate. The midpoint of AB will trace a circle.

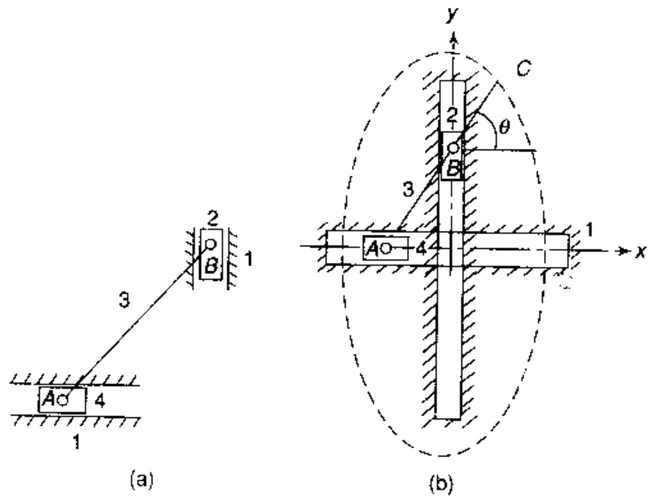

Fig. 1.58

Let at any instant, the link 3 make angle θ with the X-axis. Considering the displacements of the sliders from the centre of the trammel,

$$x = BC\cos\theta \text{ and } y = AC\sin\theta$$

$$\therefore \quad \frac{x}{BC} = \cos\theta \text{ and } \frac{y}{AC} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(BC)^2}+\frac{y^2}{(AC)^2}=\cos^2\theta+\sin^2\theta=1$$

This is the equation of an ellipse. Therefore, the path traced by C is an ellipse with the semi-major and semi-minor axes being equal to AC and BC respectively.

When C is the midpoint of AB; $AC = BC$,

and

$$\frac{x^2}{(BC)^2}+\frac{y^2}{(AC)^2}=1 \quad \text{or} \quad x^2+y^2=(AC)^2$$

which is the equation of a circle with AC (=BC) as the radius of the circle.

Second Inversion

If any of the slide-blocks of the first inversion is fixed, the second inversion of the double-slider-crank chain is obtained. When the link 4 is fixed, the end B of the crank 3 rotates about A and the link 1 reciprocates in the horizontal direction.

Application Scotch yoke

Scotch Yoke A scotch-yoke mechanism (Fig. 1.59) is used to convert the rotary motion into a sliding motion. As the crank 3 rotates, the horizontal portion of the link 1 slides or reciprocates in the fixed link 4.

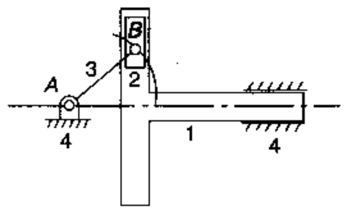

Fig. 1.59

Third Inversion

This inversion is obtained when the link 3 of the first inversion is fixed and the link 1 is free to move.

The rotation of the link 1 has been shown in Fig. 1.60 in which the full lines show the initial position. With rotation of the link 4 through 45° in the clockwise direction, the links 1 and 2 rotate through the same angle whereas the midpoint of the link 1 rotates through 90° in a circle with the length of link 3 as diameter. Thus, the angular velocity of the midpoint of the link 1 is twice that of links 2 and 4.

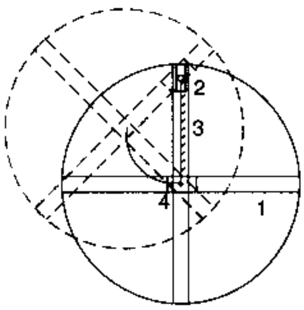

Fig. 1.60

The sliding velocity of the link 1 relative to the link 4 will be maximum when the midpoint of the link 1 is at the axis of the link 4. In this position, the sliding velocity is equal to the tangential velocity of the midpoint of the link 1.

Maximum sliding velocity = tangential velocity of midpoint of the link 1
= angular velocity of midpoint of the link 1 × radius
= (2 × angular velocity of the link 4) × (distance between axes of links 2 and 4)/2
= angular velocity of link 4 × distance between axes of links 2 and 4

The sliding velocity of the link 1 relative to the link 4 is zero when the midpoint of 1 is on the axis of the link 2.

Application Oldham's coupling

Oldham's Coupling If the rotating links 2 and 4 of the mechanism are replaced by two shafts, one can act as the driver and the other as the driven shaft with their axes at the pivots of links 2 and 4.

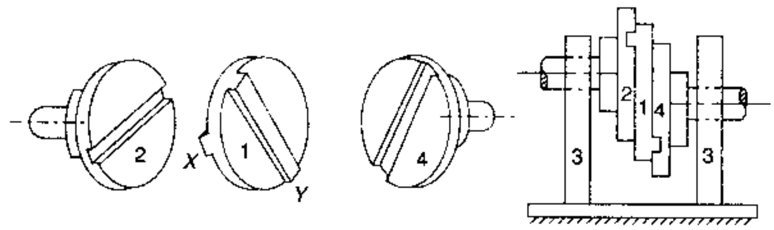

Fig. 1.61

Figure 1.61 shows an actual Oldham's coupling which is used to connect two parallel shafts when the distance between their axes is small. The two shafts have flanges at the ends and are supported in the fixed bearings representing the link 3. In the flange 2, a slot is cut in which the tongue *X* of the link 1 is fitted and has a sliding motion. Link 1 is made circular and has another tongue *Y* at right angles to the first and which fits in the recess of the flange of the shaft 4. Thus, the intermediate link 1 slides in the two slots in the two flanges while having the rotary motion.

As mentioned earlier, the midpoint of the intermediate piece describes a circle with distance between the axes of the shafts as diameter. The maximum sliding velocity of each tongue in the slot will be the peripheral velocity of the midpoint of the intermediate disc along the circular path.

Maximum sliding velocity = peripheral velocity along the circular path

= angular velocity of shaft × distance between shafts

Example 1.14 ***The distance between two parallel shafts is 18 mm and they are connected by an Oldham's coupling. The driving shaft revolves at 160 rpm. What will be the maximum speed of sliding of the tongue of the intermediate piece along its groove?***

Solution $\omega = \dfrac{2\pi \times 160}{60} = 16.75 \text{ rad/s}$

Maximum velocity of sliding = $\omega \times d$

$= 16.75 \times 0.018$

$= 0.302$ m/s

1.18 MISCELLANEOUS MECHANISMS

Snap-Action Mechanisms

The mechanisms used to overcome a large resistance of a member with a small driving force are known as *snap action* or *toggle* mechanisms. They find their use in a variety of machines such as stone crushers, embossing presses, switches, etc. Figure 1.62(a) shows such a type of mechanism in which links of equal lengths 4 and 5 are connected by a pivoted joint at *B*. Link 4 is free to oscillate about the pivot *C* and the link 5 is connected to a sliding link 6. Link 3 joins links 4 and 5. When force is applied at the point *B* through the link 3, the angle α decreases and links 4 and 5 tend to become collinear. At this instant, the force is greatly multiplied at *B*, i.e., a very small force is required to overcome a great resistance *R* at the slider. This is because a large movement at *B* produces a relatively slight displacement of the slider at *D*. As the angle α approaches zero, reaction at the pivot becomes equal to *R* and for force balance in the link *BC* or *BD*,

$$\frac{F}{2\sin\alpha} = \frac{R}{\cos\alpha}$$

or $$2\tan\alpha = \frac{F}{R}$$

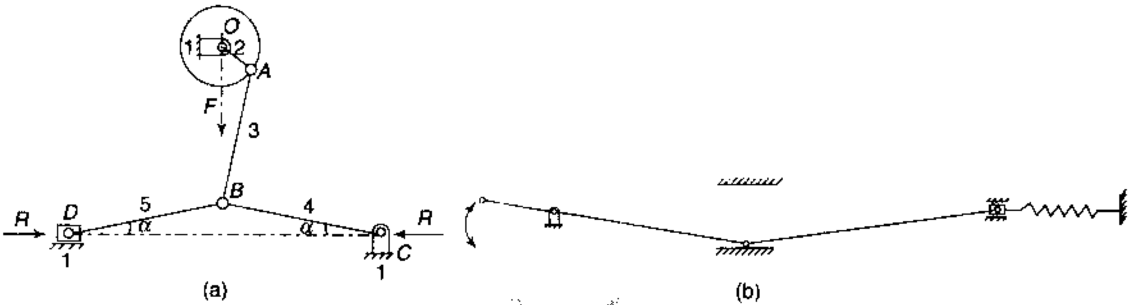

Fig. 1.62

As $\alpha \to 0$, $\tan\alpha \to 0$. Thus for a small value of the force F, R approaches infinity. In a stone crusher, a large resistance at D is overcome with a small force F in this way. Figure 1.62(b) shows another such mechanism.

Indexing Mechanisms

An *indexing mechanism* serves the purpose of dividing the periphery of a circular piece into a number of equal parts. Indexing is generally done on gear cutting or milling machines.

An indexing mechanism consists of an index head in which a spindle is carried in a headstock [Fig. 1.63(a)]. The work to be indexed is held either between centres or in a chuck attached to the spindle. A 40-tooth worm wheel driven by a single-threaded right-hand worm is also fitted to the spindle. At the end of the worm shaft an adjustable index crank with a handle and a plunger pin is also fitted. The plunger pin can be made to fit into any hole in the index plate which has a number of circles of equally spaced holes as shown in Fig. 1.63(b). An index head is usually provided with a number of interchangeable index plates to cover a wide range of work. However, the figure shows only the circle of 17 holes for sake of clarity.

As the worm wheel has 40 teeth, the number of revolutions of the index crank required to make one revolution of the work is also 40. The number of revolutions of the crank, needed for a proper division of the work into the desired number of divisions, can be calculated as follows:

- If a work is to be divided into 40 divisions, the crank should be given one complete revolution; if 20 divisions, two revolutions for each division, and so on.
- If the work is to be divided into 160 divisions, obviously the crank should be rotated through one-fourth of a rotation. For such cases, an index plate with a number of holes divisible by 4 such as with 16 or 20 holes can be chosen.
- If the work is to be divided into 136 parts, the use of the index plate will be essential since the rotation of the crank for each division will be 40/136 or five-seventeenth of a turn. Thus, a plate with 17 holes is selected in this case. To obviate the necessity of counting the holes at each partial turn of the crank, an index sector with two arms which can be set and clamped together at any angle is also available. In this case, this can be set to measure off 5 spaces. Starting with the crankpin in the hole a, a cut would be made in the work. The crank is rotated and the pin is made to enter into the hole b, 5 divisions apart and a second cut is made in the work. In a similar way, a third cut is made by rotating the crank again through five divisions with the help of an index sector, and so on. Usually, index tables are provided to ascertain the number of turns of the crank and the number of holes for the given case.

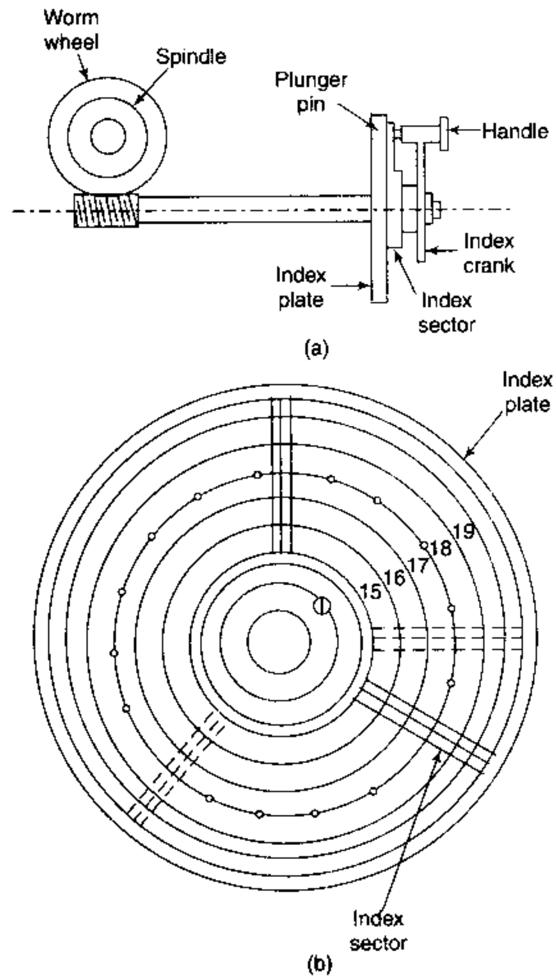

Fig. 1.63

Index plate of an indexing mechanism

Summary

1. *Kinematics* deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions whereas *dynamics* involves the calculation of forces impressed upon different parts of a mechanism.
2. *Mechanism* is a combination of a number of rigid bodies assembled in such a way that the motion of one causes constrained and predictable motion of the others whereas a *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of useful work.
3. There are three types of constrained motion: *completely constrained, incompletely constrained* and *successfully constrained.*
4. A *link* is a resistant body or a group of resistant bodies with rigid connections preventing their

relative movement. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.

5. A *kinematic pair* or simply a pair is a joint of two links having relative motion between them.
6. A pair of links having surface or area contact between the members is known as a *lower pair* and a pair having a point or line contact between the links, a *higher pair*.
7. When the elements of a pair are held together mechanically, it is known as a *closed pair*. The two elements are geometrically identical. When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an *unclosed pair*.
8. Usual types of joints in a chain are binary joint, ternary joint and quaternary joint
9. *Degree of freedom of a pair* is defined as the number of independent relative motions, both translational and rotational, a pair can have.
10. A *kinematic chain* is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite.
11. A *redundant chain* does not allow any motion of a link relative to the other.
12. A *linkage* or *mechanism* is obtained if one of the links of a kinematic chain is fixed to the ground.
13. *Degree of freedom of a mechanism* indicates how many inputs are needed to have a constrained motion of the other links.
14. *Kutzback's criterion* for the degree of freedom of plane mechanisms is

    $F = 3(N-1) - 2P_1 - 1P_2$

15. *Gruebler's criterion* for degree of freedom of plane mechanisms with single-degree of freedom joints only is

    $F = 3(N-1) - 2P_1$

16. *Author's criterion* for degree of freedom and the number of joints of plane mechanisms with turning pairs is

    $F = N - (2L + 1)$

    $P_1 = N + (L - 1)$

17. In a four-link mechanism, a link that makes a complete revolution is known as a *crank*, the link opposite to the fixed link is called the *coupler* and the fourth link is called a *lever* or *rocker* if it oscillates or another crank, if it rotates.
18. In a Watts six-bar chain, the ternary links are direct connected whereas in a Stephenson's six-bar chain, they are not direct connected.
19. If a system has one or more links which do not introduce any extra constraint, it is known as *redundant link* and is not counted to find the degree of freedom.
20. If a link of a mechanism can be moved without causing any motion to the rest of the links of the mechanism, it is said to have a *redundant degree of freedom*.
21. The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant.
22. The angle μ between the output link and the coupler is known as *transmission angle*.
23. Different mechanisms obtained by fixing different links of a kinematic chain are known as its *inversions*.
24. The mechanisms used to overcome a large resistance of a member with a small driving force are known as *snap action* or *toggle* mechanisms.
25. An *indexing mechanism* serves the purpose of dividing the periphery of a circular piece into a number of equal parts.

Exercises

1. Distinguish between
   (i) mechanism and machine
   (ii) analysis and synthesis of mechanisms
   (iii) kinematics and dynamics
2. Define: kinematic link, kinematic pair, kinematic chain.
3. What are rigid and resistant bodies? Elaborate.
4. How are the kinematic pairs classified? Explain with examples.
5. Differentiate giving examples:
   (i) lower and higher pairs
   (ii) closed and unclosed pairs
   (iii) turning and rolling pairs
6. What do you mean by degree of freedom of a

kinematic pair? How are pairs classified? Give examples.

7. Discuss various types of constrained motion.
8. What is a redundant link in a mechanism?
9. How do a Watt's six-bar chain and Stephenson's six-bar chain differ?
10. What is redundant degree of freedom of a mechanism?
11. What are usual types of joints in a mechanism?
12. What is the degree of freedom of a mechanism? How is it determined?
13. What is Kutzback's criterion for degree of freedom of plane mechanisms? In what way is Gruebler's criterion different from it?
14. How are the degree of freedom and the number of joints in a linkage can be found when the number of links and the number of loops in a kinematic chain are known?
15. What is meant by equivalent mechanisms?
16. Define Grashof's law. State how is it helpful in classifying the four-link mechanisms into different types.
17. Why are parallel-crank four-bar linkage and deltoid linkage considered special cases of four-link mechanisms?
18. Define mechanical advantage and transmission angle of a mechanism.
19. Describe various inversions of a slider-crank mechanism giving examples.
20. What are quick-return mechanisms? Where are they used? Discuss the functioning of any one of them.
21. How are the Whitworth quick-return mechanism and crank and slotted-lever mechanism different from each other?
22. Enumerate the inversions of a double-slider-crank chain. Give examples.
23. Describe briefly the functions of elliptical trammel and scotch yoke.
24. In what way is Oldham's coupling useful in connecting two parallel shafts when the distance between their axes is small?
25. What are snap-action mechanisms? Give examples.
26. What is an indexing mechanism? Describe how it is used to divide the periphery of a circular piece into a number of equal parts
27. For the kinematic linkages shown in Fig. 1.64, find the degree of freedom (F).

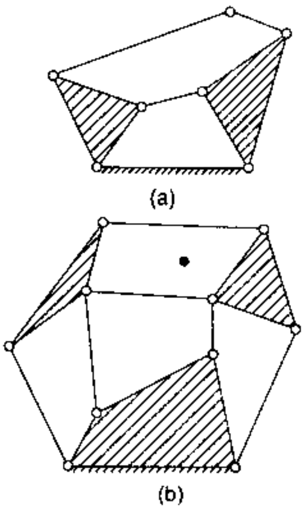

Fig. 1.64

[(a) 1 (b) 0]

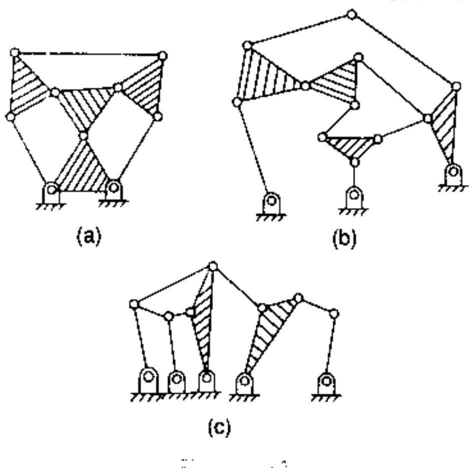

Fig. 1.65

28. For the kinematic linkages shown in Fig 1.65, find the number of binary links (N_b), ternary links (N_t), other links (N_o), total links N, loops L, joints or pairs (P_1), and degree of freedom (F).

[(a) $N_b = 3$; $N_t = 4$; $N_o = 0$; $N = 7$; $L = 3$; $P_1 = 9$; $F = 0$

(b) $N_b = 7$; $N_t = 5$; $N_o = 0$; $N = 12$; $L = 4$; $P_1 = 15$; $F = 3$

(c) $N_b = 8$; $N_t = 2$; $N_o = 1$, $N = 11$; $L = 5$; $P_1 = 15$; $F = 0$]

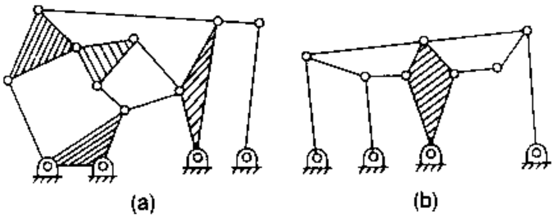

Fig. 1.66

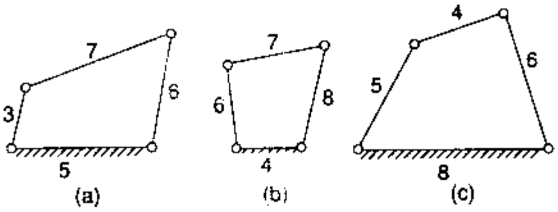

Fig. 1.67

29. Show that the linkages shown in Fig. 1.66 are structures. Suggest some changes to make them mechanisms having one degree of freedom. The number of links should not be changed by more than ±1.

30. A linkage has 14 links and the number of loops is 5. Calculate its
    - (i) degrees of freedom
    - (ii) number of joints
    - (iii) maximum number of ternary links that can be had.

    Assume that all the pairs are turning pairs.

    (3; 18; 8)

31. Figure 1.67 shows some four-link mechanisms in which the figures indicate the dimensions in standard units of length. Indicate the type of each mechanism, whether it is crank-rocker or double-crank or double-rocker.

    [(a) crank-rocker (b) double-crank (c) double-rocker]

32. A crank-rocker mechanism *ABCD* has the dimensions *AB* = 30 mm, *BC* = 90 mm, *CD* = 75 mm and *AD* (fixed link) = 100 mm. Determine the maximum and the minimum values of the transmission angle. Locate the toggle positions and indicate the corresponding crank angles and the transmission angles.

    (103°, 49°, θ = 228°, μ = 92°, θ = 38.5°, μ = 56°)

2 VELOCITY ANALYSIS

Introduction

As mentioned in the first chapter, analysis of mechanisms is the study of motions and forces concerning their different parts. The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as the angular velocities of the links. The velocity analysis is the prerequisite for acceleration analysis which further leads to force analysis of various links of a mechanism. To facilitate such study,.a machine or a mechanism is represented by a skeleton or a line diagram, commonly knows as a *configuration diagram*.

Velocities and accelerations in machines can be determined either analytically or graphically. With the invention of calculators and computers, it has become convenient to make use of analytical methods. However, a graphical analysis is more direct and is accurate to an acceptable degree and thus cannot be neglected. This chapter is mainly devoted to the study of graphical methods of velocity analysis. Two methods of graphical approach, namely, relative velocity method and instantaneous centre method are discussed. The algebraic methods are also discussed in brief. The analytical approach involving the use of calculators and computers will be discussed in Chapter 4.

2.1 ABSOLUTE AND RELATIVE MOTIONS

Strictly speaking, all motions are relative since an arbitrary set of axes or planes is required to define a motion. Usually, the earth is taken to be a fixed reference plane and all motions relative to it are termed absolute motions.

If a train moves in a particular direction, the motion of the train is referred as the absolute motion of the train or motion of the train relative to the earth. Now, suppose a man moves inside the train. Then, the motion of the man will be described in two different ways with different meanings:

1. Motion of the man relative to the train—it is equivalent to the motion of the man assuming the train to be stationary.
2. Motion of the man or absolute motion of the man or motion of the man relative to the earth = motion of man relative to the train + Motion of train relative to the earth.

2.2 VECTORS

Problems involving relative motions are conveniently solved by the use of vectors. A vector is a line which represents a vector quantity such as force, velocity, acceleration, etc.

Characteristics of a Vector

1. Length of the vector **ab** (Fig. 2.1) drawn to a convenient scale, represents the magnitude of the quantity (written as *ab*).

Direction of the line is parallel to the direction in which the quantity acts.

The initial end **a** of the line is the tail and the final end **b**, the head. An arrowhead on the line indicates the direction-sense of the quantity which is always from the tail to the head, i.e., **a** to **b**.

If the sense is as shown in Fig. 2.1(a), the vector is read as **ab** and if the sense is opposite [Fig. 2.1 (b)], the vector is read as **ba**. This implies that **ab** = − **ba**

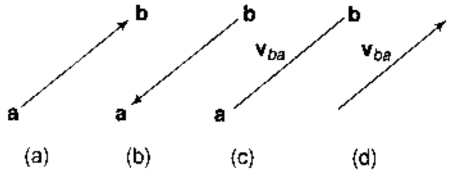

Fig. 2.1

2. Vector **ab** may also represent a vector quantity of a body B relative to a body A such as velocity of B relative to A.

   If the body A is fixed, **ab** represents the absolute velocity of B. If both the bodies A and B are in motion, the velocity of B relative to A means the velocity of B assuming the body A to be fixed for the moment.

   The vector **ab** can also be shown as $\mathbf{v}_{ba}$ [Fig. 2.1(c)], meaning the velocity of B relative to A provided a and b are indicated at the ends or an arrowhead is put on the vector [Fig. 2.1(d)].
3. Vector **ab** may also represent a vector quantity of a point B relative to a point A in the same body.

   If a vector $\mathbf{v}_{ba}$ or **ab** represents the velocity of B relative to A, the same vector in the opposite sense represents the velocity of A relative to B and will be read as $\mathbf{v}_{ab}$ or **ba**.

2.3 ADDITION AND SUBTRACTION OF VECTORS

Let

$\mathbf{v}_{ao}$ = velocity of A relative to O

$\mathbf{v}_{ba}$ = velocity of B relative to A

$\mathbf{v}_{bo}$ = velocity of B relative to O

The law of vector addition states that the velocity of B relative to O is equal to the vectorial sum of the velocity of B relative to A and the velocity of A relative to O.

Velocity of B relative to O = velocity of B relative to A + velocity of A relative to O (2.1)

i.e.
$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$
$$= \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or
$$\mathbf{ob} = \mathbf{oa} + \mathbf{ab}$$

Take the vector **oa** and place the vector **ab** at the end of the vector **oa**. Then **ob** is given by the closing side of the two vectors (Fig. 2.2).

Note that the arrows of the two vectors to be added are in the same order and that of the resultant is in the opposite order.

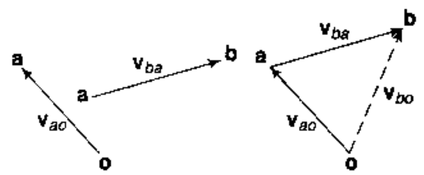

Fig. 2.2

Any number of vectors can be added as follows:

1. Take the first vector.
2. At the end of the first vector, place the beginning of the second vector.

3. At the end of the second vector, place the beginning of the third vector, and so on.
4. Joining of the beginning of the first vector and the end of the last vector represents the sum of the vectors. Figure 2.3 shows the addition of four vectors.

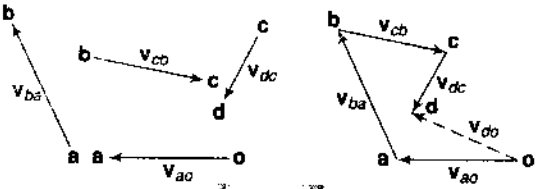

Fig. 2.3

$$\mathbf{v}_{do} = \mathbf{v}_{dc} + \mathbf{v}_{cb} + \mathbf{v}_{ba} + \mathbf{v}_{ao} \tag{2.2}$$
$$= \mathbf{v}_{ao} + \mathbf{v}_{ba} + \mathbf{v}_{cb} + \mathbf{v}_{dc}$$
$$\mathbf{od} = \mathbf{oa} + \mathbf{ab} + \mathbf{bc} + \mathbf{cd}$$

Equation 2.1 may be written as,

Vel. of B rel. to A = Vel. of B rel. to O – Vel. of A rel. to O (2.3)

$$\mathbf{v}_{ba} = \mathbf{v}_{bo} - \mathbf{v}_{ao}$$

or $$\mathbf{ab} = \mathbf{ob} - \mathbf{oa}$$

This shows that in Fig. 2.2, **ab** also represents the subtraction of **oa** from **ob** [Fig. 2.4(a)]

Also $$\mathbf{v}_{ab} = -\mathbf{v}_{ba} = \mathbf{v}_{ao} - \mathbf{v}_{bo}$$

or $$\mathbf{ba} = \mathbf{oa} - \mathbf{ob}$$

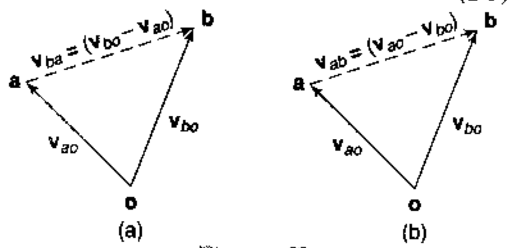

Fig. 2.4

This has been shown in Fig. 2.4 (b).

Thus, the difference of two vectors is given by the closing side of a triangle, the other two sides of which are formed by placing the two vectors tail to tail, the sense being towards the vector quantity from which the other is subtracted.

2.4 MOTION OF A LINK

Let a rigid link OA, of length r, rotate about a fixed point O with a uniform angular velocity ω rad/s in the counter-clockwise direction [Fig.2.5 (a)]. OA turns through a small angle $\delta\theta$ in a small interval of time δt. Then A will travel along the arc AA' as shown in [Fig.2.5(b)].

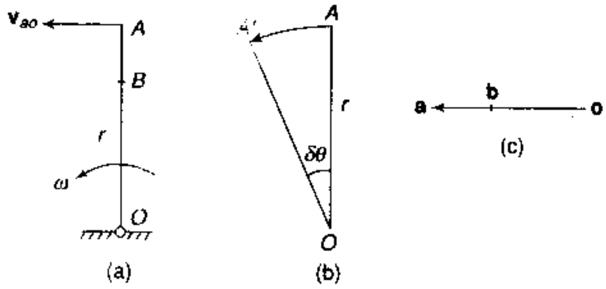

Fig. 2.5

Velocity of A relative to $O = \dfrac{\text{Arc}AA'}{\delta t}$

or
$$\mathbf{v}_{ao} = \frac{r\delta\theta}{\delta t}$$

In the limits, when $\delta t \to 0$

$$\mathbf{v}_{ao} = r\frac{d\theta}{dt} \tag{2.4}$$
$$= r\omega$$

The direction of v_{ao} is along the displacement of A. Also, as δt approaches zero ($\delta t \to 0$), AA' will be perpendicular to OA. Thus, velocity of A is ωr and is perpendicular to OA. This can be represented by a vector **oa** (Fig. 2.5 c). The fact that the direction of the velocity vector is perpendicular to the link also emerges from the fact that A can neither approach nor recede from O and thus, the only possible motion of A relative to O is in a direction perpendicular to OA.

Consider a point B on the link OA.

Velocity of $B = \omega \,.\, OB$ perpendicular to OB

If **ob** represents the velocity of B, it can be observed that

$$\frac{\mathbf{ob}}{\mathbf{oa}} = \frac{\omega OB}{\omega OA} = \frac{OB}{OA} \tag{2.5}$$

i.e., **b** divides the velocity vector in the same ratio as B divides the link.

Remember, the velocity vector v_{ao} [Fig. 2.5(c)] represents the velocity of A at a particular instant. At other instants, when the link OA assumes another position, the velocity vectors will have their directions changed accordingly.

Also, the magnitude of the instantaneous linear velocity of a point on a rotating body is proportional to its distance from the axis of rotation.

2.5 FOUR-LINK MECHANISM

Figure 2.6(a) shows a four-link mechanism (quadric-cycle mechanism) $ABCD$ in which AD is the fixed link and BC is the coupler. AB is the driver rotating at an angular speed of ω rad/s in the clockwise direction if it is a crank or moving at this angular velocity at this instant if it is a rocker. It is required to find the absolute velocity of C (or velocity of C relative to A).

Writing the velocity vector equation,

$$\text{Vel. of } C \text{ rel. to } A = \text{Vel. of } C \text{ rel. to } B + \text{vel. of } B \text{ rel. to } A$$

$$\mathbf{v}_{ca} = \mathbf{v}_{cb} + \mathbf{v}_{ba} \tag{2.6}$$

The velocity of any point relative to any other point on a fixed link is always zero. Thus, all the points on a fixed link are represented by one point in the velocity diagram. In Fig. 2.6(a), the points A and D, both lie on the fixed link AD. Therefore, the velocity of C relative to A is the same as velocity of C relative to D.

Equation (2.6) may be written as,

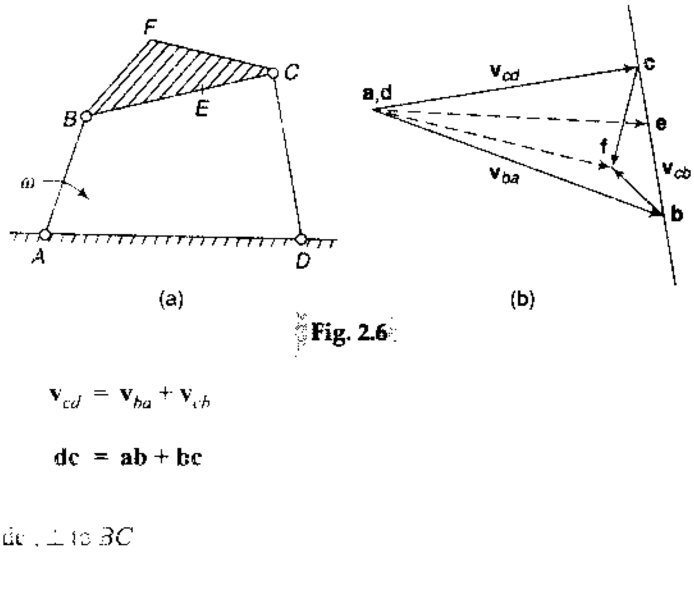

Fig. 2.6

$$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

or

$$\mathbf{dc} = \mathbf{ab} + \mathbf{bc}$$

where $\mathbf{v}_{ba}$ or $\mathbf{ab} = \omega AB$; $\perp$ to AB

$\mathbf{v}_{cb}$ or **bc** is unknown in magnitude ; $\perp$ to BC

$\mathbf{v}_{cd}$ or **dc** is unknown in magnitude ; ⊥ to DC

The velocity diagram is constructed as follows:

1. Take the first vector **ab** as it is completely known.
2. To add vector **bc** to **ab**, draw a line ⊥ BC through **b**, of any length. Since the direction-sense of **bc** is unknown, it can lie on either side of **b**. A convenient length of the line can be taken on both sides of **b**.
3. Through **d**, draw a line ⊥ DC to locate the vector **dc**. The intersection of this line with the line of vector **bc** locates the point **c**.
4. Mark arrowheads on the vectors **bc** and **dc** to give the proper sense. Then **dc** is the magnitude and also represents the direction of the velocity of C relative to A (or D). It is also the absolute velocity of the point C (A and D being fixed points).
5. Remember that the arrowheads on vector **bc** can be put in any direction because both ends of the link BC are movable. If the arrowhead is put from **c** to **b**, then the vector is read as **cb**. The above equation is modified as

$$\mathbf{dc} = \mathbf{ab} - \mathbf{cb} \qquad (\mathbf{bc} = -\,\mathbf{cb})$$

or

$$\mathbf{dc} + \mathbf{cb} = \mathbf{ab}$$

Intermediate Point

The velocity of an intermediate point on any of the links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides the link. For point E on the link BC,

$$\frac{\mathbf{be}}{\mathbf{bc}} = \frac{BE}{BC}$$

ae represents the absolute velocity of E.

Offset Point

Write the vector equation for point F,

$$\mathbf{v}_{fb} + \mathbf{v}_{ba} = \mathbf{v}_{fc} + \mathbf{v}_{cd}$$

or

$$\mathbf{v}_{ba} + \mathbf{v}_{fb} = \mathbf{v}_{cd} + \mathbf{v}_{fc}$$

or

$$\mathbf{ab} + \mathbf{bf} = \mathbf{dc} + \mathbf{cf}$$

The vectors $\mathbf{v}_{ba}$ and $\mathbf{v}_{cd}$ are already there on the velocity diagram.

$\mathbf{v}_{fb}$ is ⊥ BF, draw a line ⊥ BF through **b**;

$\mathbf{v}_{fc}$ is ⊥ CF, draw a line ⊥ CF through **c**;

The intersection of the two lines locates the point **f**.

af or **df** indicates the velocity of F relative to A (or D) or the absolute velocity of F.

2.6 VELOCITY IMAGES

Note that in Fig. 2.6, the triangle **bfc** is similar to the triangle BFC in which all the three sides **bc**, **cf** and **fb** are perpendicular to BC, CF and FB respectively. The triangles such as **bcf** are known as velocity images and are found to be very helpful devices in the velocity analysis of complicated shapes of the linkages. Thus, any offset point on a link in the configuration diagram can easily be located in the velocity diagram by drawing the velocity image. While drawing the velocity images, the following points should be kept in mind:

1. The velocity image of a link is a scaled reproduction of the shape of the link in the velocity diagram from the configuration diagram, rotated bodily through 90° in the direction of the angular velocity.
2. The order of the letters in the velocity image is the same as in the configuration diagram.
3. In general, the ratio of the sizes of different images to the sizes of their respective links is different in the same mechanism.

2.7 ANGULAR VELOCITY OF LINKS

1. Angular Velocity of BC

(a) Velocity of C relative to *B*, v_{cb} = **bc** (Fig. 2.6)

Point *C* relative to *B* moves in the direction-sense given by $\mathbf{v}_{cb}$ (upwards). Thus, *C* moves in the counter-clockwise direction about *B*.

$$\mathbf{v}_{cb} = \omega_{cb} \times BC = \omega_{cb} \times CB$$

$$\omega_{cb} = \frac{v_{cb}}{CB}$$

(b) Velocity of *B* relative to *C*, $\mathbf{v}_{bc}$ = **cb**

B relative to *C* moves in a direction-sense given by v_{bc} (downwards, opposite to **bc**), i.e., *B* moves in the counter-clockwise direction about *C* with magnitude ω_{bc} given by

$$\frac{v_{bc}}{BC}$$

It can be seen that the magnitude of $\omega_{cb} = \omega_{bc}$ as $v_{cb} = v_{bc}$ and the direction of rotation is the same. Therefore, angular velocity of a link about one extremity is the same as the angular velocity about the other. In general, the angular velocity of link *BC* is ω_{bc} (= ω_{cb}) in the counter-clockwise direction.

2. Angular Velocity of CD

Velocity of *C* relative to *D*,

$$v_{cd} = \mathbf{dc}$$

It is seen that *C* relative to *D* moves in a direction-sense given by v_{cd} or *C* moves in the clockwise direction about *D*.

$$\omega_{cd} = \frac{v_{cd}}{CD}$$

2.8 VELOCITY OF RUBBING

Figure 2.7 shows two ends of the two links of a turning pair. A pin is fixed to one of the links whereas a hole is provided in the other to fit the pin. When joined, the surface of the hole of one link will rub on the surface of the pin of the other link. The velocity of rubbing of the two surfaces will depend upon the angular velocity of a link relative to the other.

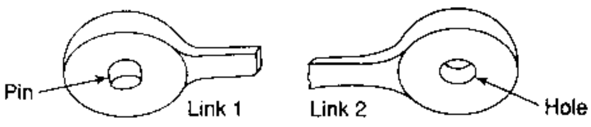

Fig. 2.7

Pin at A (Fig. 2.6a)

The pin at A joins links AD and AB. AD being fixed, the velocity of rubbing will depend upon the angular velocity of AB only.

Let r_a = radius of the pin at A

Then velocity of rubbing = $r_a . \omega$

Pin at D

Let r_d = radius of the pin at D

Velocity of rubbing = $r_d . \omega_{cd}$

Pin at B

$\omega_{ba} = \omega_{ab} = \omega$ clockwise

$\omega_{bc} = \omega_{cb} = \frac{v_{bc}}{BC}$ counter-clockwise

Since the directions of the two angular velocities of links AB and BC are in the opposite directions, the angular velocity of one link relative to the other is the sum of the two velocities.

Let r_b = radius of the pin at B

Velocity of rubbing = $r_b (\omega_{ab} + \omega_{bc})$

Pin at C

$\omega_{bc} = \omega_{cb}$ counter-clockwise

$\omega_{dc} = \omega_{cd}$ clockwise

Let r_c = radius of the pin at C

Velocity of rubbing = $r_c (\omega_{bc} + \omega_{dc})$

In case it is found that the angular velocities of the two links joined together are in the same direction, the velocity of rubbing will be the difference of the angular velocities multiplied by the radius of the pin.

2.9 SLIDER-CRANK MECHANISM

Figure 2.8(a) shows a slider-crank mechanism in which OA is the crank moving with uniform angular velocity ω rad/s in the clockwise direction. At point B, a slider moves on the fixed guide G. AB is the coupler joining A and B. It is required to find the velocity of the slider at B.

Writing the velocity vector equation,

Vel. of B rel. to O = Vel. of B rel. to A + Vel. of A rel. to O

$$v_{bo} = v_{ba} + v_{ao}$$

$$v_{bg} = v_{ao} + v_{ba}$$

or $$\mathbf{gb = oa + ab}$$

$\mathbf{v}_{bo}$ is replaced by v_{bg} as O and G are two points on a fixed link

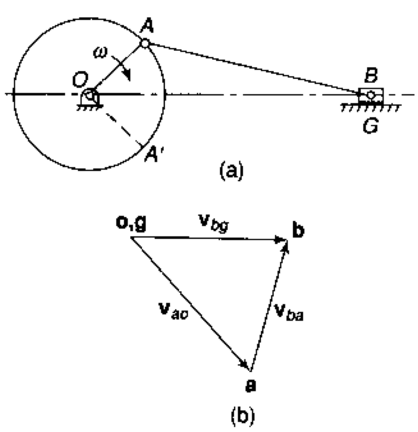

Fig. 2.8

with zero relative velocity between them.

Take the vector $\mathbf{v}_{ao}$ which is completely known.

$\mathbf{v}_{ao} = \omega . OA$; $\perp$ to OA

$\mathbf{v}_{ba}$ is $\perp$ AB, draw a line $\perp$ AB through **a**;

Through **g** (or **a**), draw a line parallel to the motion of B (to locate the vector $\mathbf{v}_{bg}$).

The intersection of the two lines locates the point **b**.

gb (or **ob**) indicates the velocity of the slider B relative to the guide G. This is also the absolute velocity of the slider (G is fixed). The slider moves towards the right as indicated by **gb.** When the crank assumes the position OA' while rotating, it will be found that the vector **gb** lies on the left of **g** indicating that B moves towards left.

For the given configuration, the coupler AB has angular velocity in the counter-clockwise direction, the magnitude being $\dfrac{v_{ba}}{BA(\text{or } AB)}$

Example 2.1 *In a four-link mechanism, the dimensions of the links are as under:*
AB = 50 mm, BC = 66 mm, CD = 56 mm and AD = 100 mm

At the instant when ∠DAB = 60°, the link AB has an angular velocity of 10.5 rad/s in the counter-clockwise direction. Determine the

(i) velocity of the point C
(ii) velocity of the point E on the link BC when BE = 40 mm
(iii) angular velocities of the links BC and CD
(iv) velocity of an offset point F on the link BC if BF = 45 mm, CF = 30 mm and BCF is read clockwise
(v) velocity of an offset point G on the link CD if CG = 24 mm, DG = 44 mm and DCG is read clockwise
(vi) velocity of rubbing at pins A, B, C and D when the radii of the pins are 30, 40, 25 and 35 mm respectively.

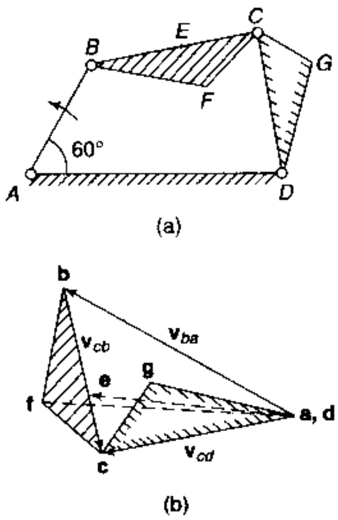

Fig. 2.9

Solution The configuration diagram has been shown in Fig. 2.9(a) to a convenient scale.

Writing the vector equation,

Vel. of C rel. to A = Vel. of C rel. to B + Vel. of B rel. to A

$$\mathbf{v}_{ca} = \mathbf{v}_{cb} + \mathbf{v}_{ba}$$

or $$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

or $$\mathbf{dc} = \mathbf{ab} + \mathbf{bc}$$

We have,

$$\mathbf{v}_{ba} = \omega_{ba} \times BA = 10.5 \times 0.05 = 0.525 \text{ m/s}$$

Take the vector $\mathbf{v}_{ba}$ to a convenient scale in the proper direction and sense [Fig. 2.9(b)].

$\mathbf{v}_{cb}$ is $\perp$ BC, draw a line $\perp$ BC through **b;**

$\mathbf{v}_{cd}$ is $\perp$ DC, draw a line $\perp$ DC through **d;**

The intersection of the two lines locates the point **c.**

Note In the velocity diagram shown in Fig. 2.9(b), arrowhead has been put on the line joining points b and c in such a way that it represents the vector for velocity of C relative to B. This satisfies the above equation. As the same equation

$$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

can also be put as

$$\mathbf{v}_{cd} + \mathbf{v}_{bc} = \mathbf{v}_{ba}$$
$$\mathbf{dc} + \mathbf{cb} = \mathbf{ab}$$

This shows that on the same line joining **b** and **c**, the arrowhead should be marked in the other direction so that it represents the vector of velocity of B relative to C to satisfy the latter equation.

Thus, it implies that in case both the ends of a link are in motion, the arrowhead may be put in either direction or no arrowhead is put at all. This is because every time it is not necessary to write the velocity equation. The velocity equation is helpful only at the initial stage for better comprehension.]

(i) v_c = **ac** (or **dc**) = 0.39 m/s

(ii) Locate the point **e** on **bc** such tha $\frac{be}{bc} = \frac{BE}{BC}$

bc = 0.34 m/s from the velocity diagram.

$$\mathbf{be} = 0.34 \times \frac{0.040}{0.066} = 0.206 \text{ m/s}$$

Therefore, v_e = **ae** (or **de**) = 0.415 m/s

(iii) $$\omega_{cb} = \frac{v_{cb}}{CB} = \frac{0.340}{0.066} = 5.15 \text{ rad/s clockwise}$$

$$\omega_{cd} = \frac{v_{cd}}{CD} = \frac{0.390}{0.056} = 6.96 \text{ rad/s}$$

counter-clockwise

(iv) $\mathbf{v}_{fb}$ is $\perp BF$, draw a line $\perp BF$ through **b**;
$\mathbf{v}_{fc}$ is $\perp CF$, draw a line $\perp CF$ through **c**;
The intersection locates the point **f**.
v_f (i.e., v_{fa} or v_{fd}) = **af** = 0.495 m/s

The point **f** can also be located by drawing the velocity image **bcf** of the triangle BCF as discussed earlier.

(v) $\mathbf{v}_{gd}$ is $\perp DG$, draw **dg** $\perp DG$ through **d**;
$\mathbf{v}_{gc}$ is $\perp CG$, draw **cg** $\perp CG$ through **c**.
The intersection locates the point **g**.
v_g = **dg** = 0.305 m/s

However, the velocity of G could be found directly since G is a point on the link DC which rotates about a fixed point D and the velocity of C is already known.

$$\frac{v_g}{v_c} = \frac{DG}{DC}$$

or

$$v_g = 0.390 \times \frac{0.044}{0.056} = 0.306 \text{ m/s}$$

The point **g** can also be located by drawing the velocity image **dcg** of the triangle dCG.

(vi) (a) ω_{ba} (or ω_{ab}) is counter-clockwise and ω_{cb} (or ω_{bc}) is clockwise,

Velocity of rubbing at pin $B = (\omega_{ab} + \omega_{cb})r_b$
$= (10.5 + 5.15) \times 0.040$
$= 0.626$ m/s

(b) ω_{dc} is counter-clockwise and ω_{bc} is clockwise,

Velocity of rubbing at the pin C
$= (\omega_{dc} + \omega_{bc})r_c$
$= (6.96 + 5.15) \times 0.025$
$= 0.303$ m/s

(c) Velocity of rubbing at the pin A
$= \omega_{ba} r_a = 10.5 \times 0.03 = 0.315$ m/s

(d) Velocity of rubbing at the pin D
$= \omega_{cd} r_d = 6.96 \times 0.035 = 0.244$ m/s

Example 2.2 *In a slider-crank mechanism, the crank is 480 mm long and rotates at 20 rad/s in the counter-clockwise direction The length of the connecting rod is 1.6 m. When the crank turns 60° from the inner-dead centre, determine the*

(i) velocity of the slider

(ii) velocity of a point E located at a distance 450 mm on the connecting rod extended

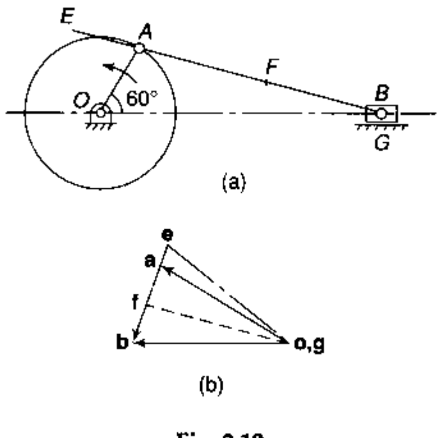

Fig. 2.10

(iii) position and velocity of a point F on the connecting rod having the least absolute velocity

(iv) angular velocity of the connecting rod

(v) velocities of rubbing at the pins of the crankshaft, crank and the cross-head having diameters 80, 60 and 100 mm respectively.

Solution Figure 2.10(a) shows the configuration diagram to a convenient scale.

$v_{ao} = \omega_{ao} \times OA = 20 \times 0.48 = 9.6$ m/s

The vector equation is $\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$

or

$$\mathbf{v}_{bg} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{bg} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or

$$\mathbf{gb} = \mathbf{oa} + \mathbf{ab}$$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale in the proper direction and sense [Fig. 2.10 (b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a**;

The slider B has a linear motion relative to the guide G. Draw a line parallel to the direction of motion of the slider through **g** (or **o**). Thus, the point **b** is located.

(i) Velocity of the slider, $v_b = \mathbf{ob} = 9.7$ m/s

(ii) Locate the point **e** on **ba** extended such that

$$\frac{\mathbf{ae}}{\mathbf{ba}} = \frac{AE}{BA}$$

ba = 5.25 m/s on measuring from the diagram.

$$\therefore \mathbf{ae} = 5.25 \times \frac{0.45}{1.60} = 1.48 \text{ m/s}$$

$v_e = \mathbf{oe} = \underline{10.2 \text{ m/s}}$

(iii) To locate a point F on the connecting rod which has the least velocity relative to the crankshaft or has the least absolute velocity, draw **of** $\perp$ **ab** through **o**.

Locate the point F on AB such the $\frac{AF}{AB} = \frac{\mathbf{af}}{\mathbf{ab}}$

or

$$AF = 1.60 \times \frac{2.76}{5.25} = 0.84 \text{ m}$$

$v_f = \mathbf{of} = \underline{9.4 \text{ m/s}}$

(iv) $\omega_{ba} = \frac{v_{ba}}{AB} = \frac{5.25}{1.60} = 3.28$ rad/s clockwise

(v) (a) Velocity of rubbing at the pin of the crankshaft (at O)

$= \omega_{ao} r_o = 20 \times 0.04 = 0.8$ m/s

$$\left(r_o \frac{80}{2} = 40 \text{ mm} \right)$$

(b) ω_{oa} is counter-clockwise and ω_{ba} is clockwise.

Velocity of rubbing at the crank pin $A = (\omega_{oa} - \omega_{ba})\, r_a$

$= (20 + 3.28) \times 0.03$

$= 0.698$ m/s

(c) At the cross-head, the slider does not rotate and only the connecting rod has the angular motion.

Velocity of rubbing at the cross-head pin at B

$= \omega_{ab} r_b = 3.28 \times 0.05 = \underline{0.164 \text{ m/s}}$

Example 2.3 *Figure 2.11a shows a mechanism in which OA = QC = 100 mm, AB = QB = 300 mm and CD = 250 mm. The crank OA rotates at 150 rpm in the clockwise direction. Determine the*

(i) velocity of the slider at D

(ii) angular velocities of links QB and AB

(iii) rubbing velocity at the pin B which is 40 mm in diameter

Solution $\omega_{ao} = \frac{2\pi \times 150}{60} = 15.7$ rad/s

$v_{ao} = \omega_{ao} \times OA = 15.7 \times 0.1 = 1.57$ m/s

The vector equation for the mechanism $OABQ$,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or $\mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$ or $\mathbf{qb} = \mathbf{oa} + \mathbf{ab}$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale in the proper direction and sense [Fig. 2.11 (b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a**;

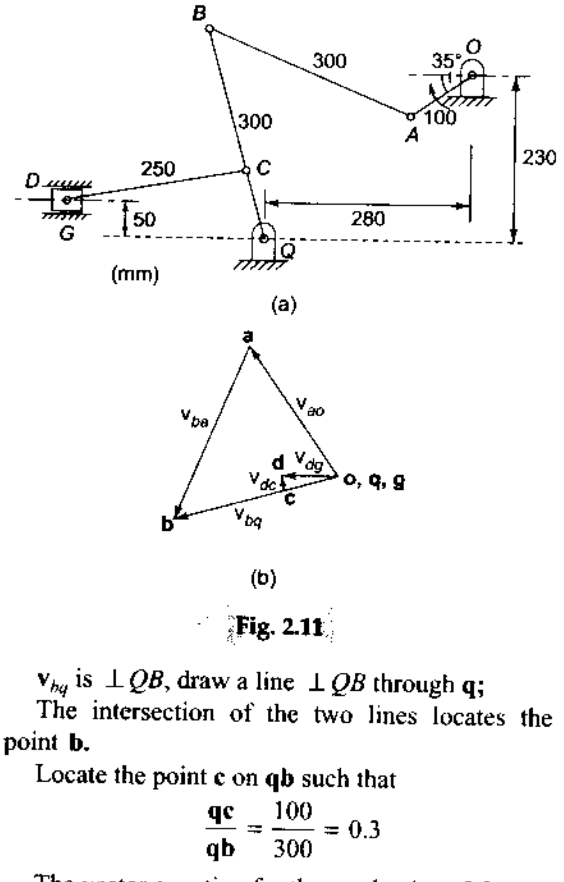

Fig. 2.11

$\mathbf{v}_{bq}$ is $\perp QB$, draw a line $\perp QB$ through **q**;

The intersection of the two lines locates the point **b**.

Locate the point **c** on **qb** such that

$$\frac{\mathbf{qc}}{\mathbf{qb}} = \frac{100}{300} = 0.3$$

The vector equation for the mechanism QCD,

$$\mathbf{v}_{dq} = \mathbf{v}_{dc} + \mathbf{v}_{cq} \quad \text{or} \quad \mathbf{v}_{dg} = \mathbf{v}_{cq} + \mathbf{v}_{dc}$$

or $\quad \mathbf{gd} = \mathbf{qc} + \mathbf{cd}$

$\mathbf{v}_{dc}$ is $\perp DC$, draw a line $\perp DC$ through **c**;

For $\mathbf{v}_{dg}$, draw a line through **g**, parallel to the line of stroke of the slider in the guide G.

The intersection of the two lines locates the point **d**.

(i) The velocity of slider at D, v_d = **gd** = 0.56 m/s

(vi) $\omega_{bq} = \frac{v_{bq}}{QB} = \frac{1.69}{0.3} = 5.63$ rad/s counter-clockwise

(vii) $\omega_{ba} = \frac{v_{ba}}{AB} = \frac{1.89}{0.3} = 6.3$ rad/s counter-clockwise

As both the links connected at B have counter-clockwise angular velocities,

velocity of rubbing at the crank pin

$B = (\omega_{ba} - \omega_{bq})\, r_b$

$= (6.3 - 5.63) \times 0.04 = 0.0268$ m/s

Example 2.4 *An engine crankshaft drives a reciprocating pump through a mechanism as shown in Fig. 2.12(a). The crank rotates in the clockwise direction at 160 rpm. The diameter of the pump piston at F is 200 mm. Dimensions of the various links are*

OA = 170 mm (crank) CD = 170 mm
AB = 660 mm DE = 830 mm
BC = 510 mm

For the position of the crank shown in the diagram, determine the

(i) velocity of the crosshead E

(ii) velocity of rubbing at the pins A, B, C and D, the diameters being 40, 30, 30 and 50 mm respectively

(iii) torque required at the shaft O to overcome a pressure of 300 kN/m² at the pump piston at F

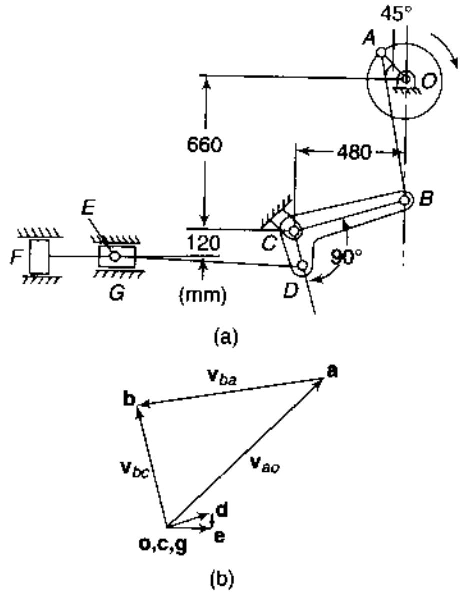

Fig. 2.12

Solution:

$$\omega_{ao} = \frac{2\pi N}{60} = \frac{2\pi \times 160}{60} = 16.76 \text{ rad/s}$$

$$v_{ao} = \omega_{ao} \times OA = 16.76 \times 0.17 = 2.85 \text{ m/s}$$

Writing the vector equation for the mechanism *OABC*,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{bc} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or

$$\mathbf{cb} = \mathbf{oa} + \mathbf{ab}$$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.12(b)]

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a**;

$\mathbf{v}_{bc}$ is $\perp BC$, draw a line $\perp BC$ through **c**.

The intersection of the two lines locates the point ***b***. Velocity of *B* relative to *C* is upwards for the given configuration. Therefore, the link *BCD* moves counter-clockwise about the pivot *C*.

$$\frac{v_{dc}}{v_{bc}} = \frac{DC}{BC}$$

or $v_{dc} = 1.71 \times \dfrac{0.17}{0.51} = 0.57$ m/s $\qquad (\perp DC)$

Writing the vector equation for the mechanism *CDE*,

$$\mathbf{v}_{ec} = \mathbf{v}_{ed} + \mathbf{v}_{dc}$$

or

$$\mathbf{v}_{eg} = \mathbf{v}_{dc} + \mathbf{v}_{ed}$$

or

$$\mathbf{ge} = \mathbf{cd} + \mathbf{de}$$

Take $\mathbf{v}_{dc}$ in the proper direction and sense from **c** assuming *D* in the configuration diagram as an offset point on link *CB*;

$\mathbf{v}_{ed}$ is $\perp DE$, draw a line $\perp DE$ through **d**.

For $\mathbf{v}_{eg}$, draw a line through **g**, parallel to the direction of motion of the slider *E* in the guide *G*.

This way the point **e** is located.

(i) The velocity of the crosshead,

v_e = **oe** = <u>0.54 m/s</u>

(ii) (a) ω_{oa} and ω_{ba} both are clockwise.

$$\omega_{ba} = \frac{\mathbf{ab}}{AB} = \frac{2.49}{0.66} = 3.77 \text{ rad/s}$$

Velocity of rubbing at the pin $A = (\omega_{oa} - \omega_{ba})\, r_a$

$= (16.76 - 3.77) \times \dfrac{0.04}{2}$

$=$ <u>0.26 m/s</u>

(b) ω_{ab} is clockwise and ω_{cd} is counter-clockwise.

$$\omega_{cb} = \frac{v_{cb}}{CB} = \frac{1.71}{0.51} = 3.35 \text{ rad/s}$$

Velocity of rubbing at $B = (\omega_{ab} + \omega_{cb})\, r_b$

$= (3.77 + 3.35) \times 0.015 \;\ldots\; (\omega_{ab} = \omega_{ba})$

$=$ <u>0.107 m/s</u>

(c) Velocity of rubbing at $C = \omega_{bc} . r_c$

$= 3.35 \times \dfrac{0.03}{2} =$ <u>0.05 m/s</u>

(d) ω_{cd} and ω_{ed} both are counter-clockwise

$\omega_{cd} = \omega_{bc} = 3.35$ rad/s ... (*BCD* is one link)

$= \omega_{ed} = \dfrac{v_{ed}}{ED} = \dfrac{0.15}{0.83} = 0.18$ rad/s

Velocity of rubbing at $D = (\omega_{cd} - \omega_{ed})\, r_d$

$= (3.35 - 0.18) \times \dfrac{0.05}{2}$

$=$ <u>0.079 m/s</u>

(iii) Work input = work output

$T.\omega = F.v$

where T = torque on the crankshaft

ω = angular velocity of the crank

F = force on the piston

v = velocity of the piston $= v_f = v_e$

Thus, neglecting losses,

$$T = \frac{F.v}{\omega} = \frac{\pi}{4}(0.02)^2 \times 300 \times 10^3 \times \frac{0.54}{16.76}$$

$=$ <u>303.66 N.m</u>

Example 2.5 *Figure 2.13(a) shows a mechanism in which OA = 300 mm, AB = 600 mm, AC = BD = 1.2 m. OD is horizontal for the given configuration. If OA rotates at 200 rpm in the clockwise direction, find*

(iv) linear velocities of C and D

(v) angular velocities of links AC and BD

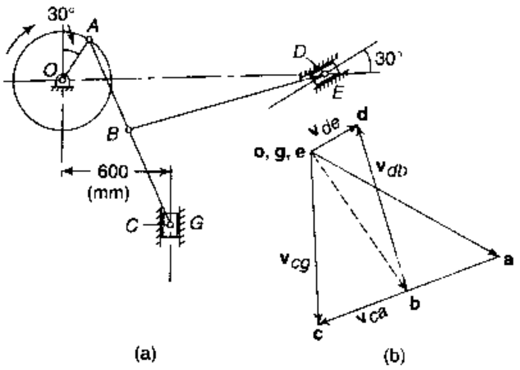

Fig. 2.13

Solution: $\omega_a = \dfrac{2\pi \times 200}{60} = 20.94$ rad/s

$$v_a = \omega_a OA = 20.94 \times 0.3 = 6.28 \text{ m/s}$$

Writing the vector equation for the mechanism *OAC*,

$$\mathbf{v}_{co} = \mathbf{v}_{ca} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{cg} = \mathbf{v}_{ao} + \mathbf{v}_{ca}$$

or

$$\mathbf{gc} = \mathbf{oa} + \mathbf{ac}$$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.13(b)].

$\mathbf{v}_{ca}$ is $\perp AC$, draw a line $\perp AC$ through **a**;

$\mathbf{v}_{cg}$ is vertical, draw a vertical line through **g** (or **o**).

The intersection of the two lines locates the point **c**. Locate the point **b** on **ac** as usual. Join **ob** which gives $\mathbf{v}_{bo}$. Writing the vector equation for the mechanism *OABD*,

$$\mathbf{v}_{do} = \mathbf{v}_{db} + \mathbf{v}_{bo}$$

or

$$\mathbf{v}_{de} = \mathbf{v}_{bo} + \mathbf{v}_{db}$$

or

$$\mathbf{ed} = \mathbf{ob} + \mathbf{bd}$$

$\mathbf{v}_{db}$ is $\perp BD$, draw a line $\perp BD$ through **b**;

For $\mathbf{v}_{de}$, draw a line through **e**, parallel to the line of stroke of the piston in the guide *E*.

The intersection locates the point **d**.

$$v_c = \mathbf{oc} = \underline{5.2 \text{ m/s}}$$

$$v_d = \mathbf{od} = \underline{1.55 \text{ m/s}}$$

$$\omega_{ac} = \omega_{ca} = \frac{v_{ca}}{AC} = \frac{5.7}{1.20} = \underline{4.75 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{bd} = \omega_{db} = \frac{v_{db}}{BD} = \frac{5.17}{1.20} = \underline{4.31 \text{ rad/s}} \text{ clockwise}$$

Example 2.6 *For the position of the mechanism shown in Fig. 2.14(a), find the velocity of the slider B for the given configuration if the velocity of the slider A is 3 m/s.*

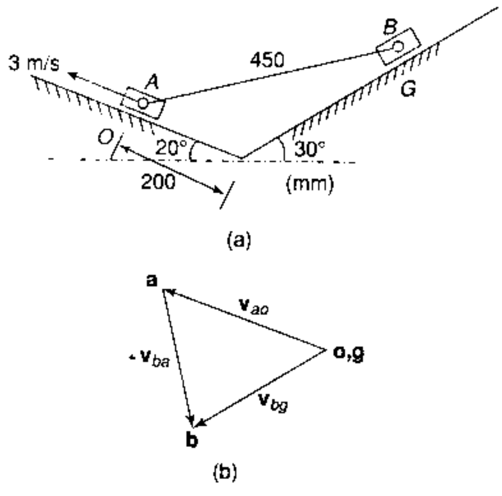

Fig. 2.14

Solution The velocity vector equation is

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or $\quad \mathbf{v}_{bg} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$

or $\quad \mathbf{gb} = \mathbf{oa} + \mathbf{ab}$

Take the vector $\mathbf{v}_{ao}$ (= 3 m/s) to a convenient scale [Fig. 2.14(b)]

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line AB through **a**;

For $\mathbf{v}_{bg}$, draw a line through **g** parallel to the line of stroke of the slider B on the guide G.

The intersection of the two lines locates the point **b**.

Velocity of B = **gb** = 2.67 m/s.

Example 2.7 *In a mechanism shown in Fig. 2.15(a), the angular velocity of the crank OA is 15 rad/s and the slider at E is constrained to move at 2.5 m/s. The motion of both the sliders is vertical and the link BC is horizontal in the position shown. Determine the*

(i) rubbing velocity at B if the pin diameter is 15 mm

(ii) velocity of slider D.

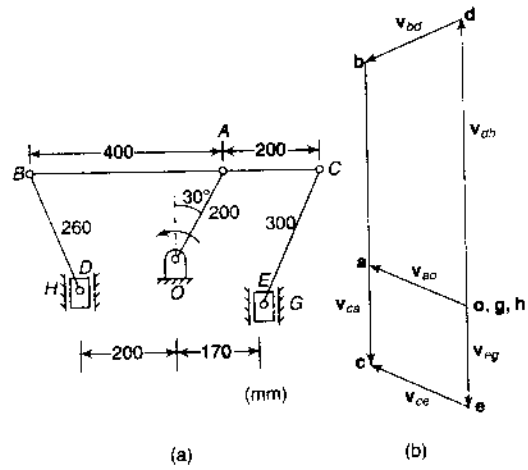

Fig. 2.15

Solution $v_a = \omega_a OA = 15 \times 0.2 = 3$ m/s

Draw the velocity diagram as follows:

- Take vector **oa** to a suitable scale (2.15b).
- Consider two points G and H on the guides of sliders E and F respectively. In the velocity diagram, the points **g** and **h** coincide with **o**. Through **g**, take a vector **ge** parallel to direction of motion of the slider E and equal to 2.5 m/s using some scale.
- Through **c** draw a line $\perp EC$ and through **a**, a line $\perp AC$, the intersection of these two lines locates the point **c**.
- Locate the point **b** on the vector **ca** so that **ca/cb** = CA/CB.
- Through **b**, draw a line $\perp BD$ and through **h**, a line parallel to direction of motion of the slider D, the intersection of these two lines locates the point **d**.

(i) Angular velocity of link BD = **bd**/BD = 2.95/0.26 = 11.3 rad/s (counter-clockwise)
Angular velocity of link BC = **bc**/BC = 8.4/0.6 = 14 rad/s (clockwise)
Thus velocity of rubbing at

$$B = (\omega_{bd} + \omega_{bc}) r_b$$
$$= (11.3 - 14) \times 0.015$$
$$= \underline{0.38 \text{ m/s}}$$

(ii) The velocity of the slider D = **hd** = <u>8.3 m/s</u>

Example 2.8 *The lengths of various links of a mechanism shown in Fig. 2.16(a) are as follows:*

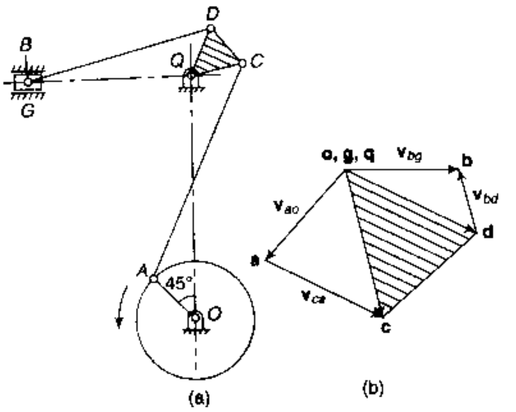

Fig. 2.16

OA = 150 mm	*CD = 125 mm*
AC = 600 mm	*BD = 500 mm*
CQ = QD = 145 mm	*OQ = 625 mm*

The crank OA rotates at 60 rpm in the counter-clockwise direction. Determine the velocity of the slider B and the angular velocity of the link BD when the crank has turned an angle of 45° with the vertical.

Solution

$$v_a = \frac{2\pi N}{60} \times OA = \frac{2\pi \times 60}{60} \times 0.15 = 0.94 \text{ m/s}$$

Take the vector $\mathbf{v}_a$, to a convenient scale [Fig. 2.16(b)] and complete the velocity diagram for the mechanism *OACQ*.

Now *CQD* is one link. Make Δ **cqd** similar to Δ *CQD* such that **cqd** reads clockwise as *CQD* is clockwise. This locates the point **d**. Complete the velocity diagram for the mechanism *QDB*.

v_b = **ob** = 0.9 m/s

$$\omega_{bd} = \frac{v_{bd}}{BD} = \frac{0.49}{0.50} = \underline{0.98 \text{ rad/s}} \text{ clockwise}$$

Example 2.9 *The configuration diagram of a wrapping machine is given in Fig. 2.17(a). The crank OA rotates at 6 rad/s clockwise. Determine the*

(i) velocity of the point P on the bell-crank lever DCP

(ii) angular velocity of the bell-crank lever DCP

(iii) velocity of rubbing at B if the pin diameter is 20 mm

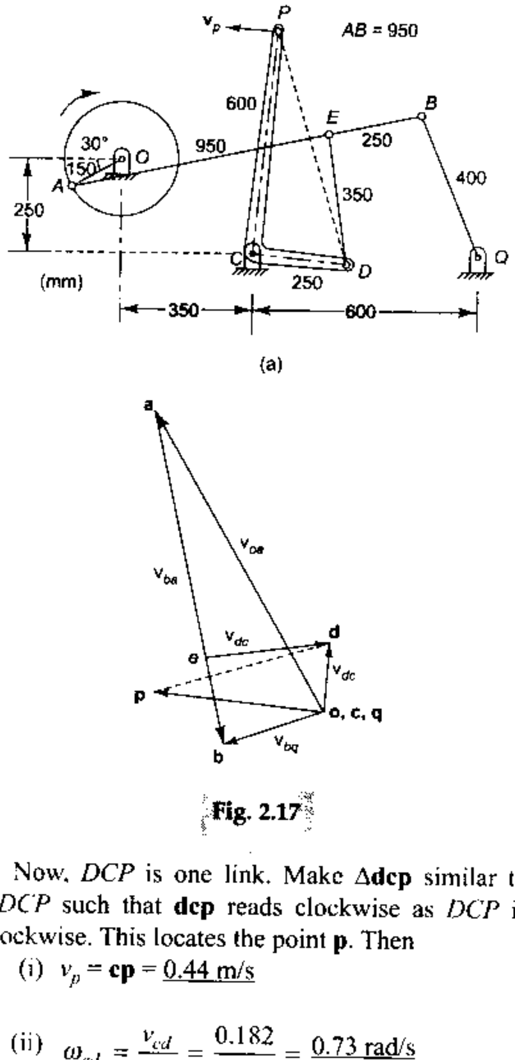

Fig. 2.17

Solution

$$v_a = 6 \times 0.15 = 0.9 \text{ m/s}$$

Take the vector $\mathbf{v}_a$, to a convenient scale [Fig. 2.17(b)] and complete the velocity diagram for the mechanism *OABQ*.

Now locate point **e** on the vector **ab**.

$\mathbf{v}_{de}$ is ⊥ *DE*, draw **de** ⊥ *DE* through **e**;

$\mathbf{v}_{dc}$ is ⊥ *CD*, draw **cd** ⊥ *CD* through **c**.

The intersection locates the point **d**.

Now, *DCP* is one link. Make Δ**dcp** similar to Δ*DCP* such that **dcp** reads clockwise as *DCP* is clockwise. This locates the point **p**. Then

(i) v_p = **cp** = 0.44 m/s

(ii) $$\omega_{cd} = \frac{v_{cd}}{CD} = \frac{0.182}{0.25} = \underline{0.73 \text{ rad/s}}$$

counter clockwise

(iii) $$\omega_{ab} = \frac{v_{ab}}{AB} = \frac{0.91}{0.95} = \underline{0.96 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{qb} = \frac{v_{qb}}{QB} = \frac{0.28}{0.4} = \underline{0.7 \text{ rad/s}}$$

counter-clockwise

Thus, velocity of rubbing at $B = (\omega_{ab} + \omega_{bq})r_b$

$= (0.96 + 0.7) \times 0.02 = \underline{0.0332 \text{ m/s}}$

Example 2.10 *Figure 2.18(a) shows an Andrew variable-stroke-engine mechanism. The lengths of the cranks OA and QB are 90 mm and 45 mm respectively. The diameters of wheels with centres O and Q are 250 mm and 120 mm respectively. Other lengths are shown in the diagram in mm. There is a rolling contact between the two wheels. If OA rotates at 100 rpm, determine the*

(i) velocity of the slider D

(ii) angular velocities of links BC and CD

(iii) torque at QB when force required at D is 3 kN

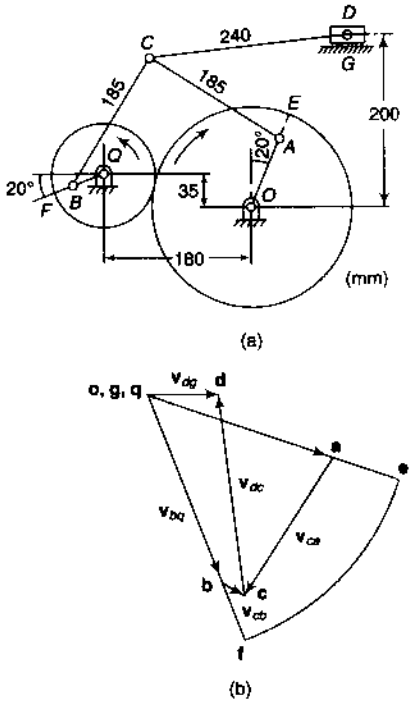

Fig. 2.18

Solution

$$v_a = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} \times 0.09 = 0.943 \text{ m/s}$$

$$v_e = v_a \frac{OE}{OA} = 0.943 \times \frac{0.125}{0.09} = 1.309 \text{ m/s}$$

$$v_f = v_e = 1.309 \text{ m/s}$$

$$v_b = v_f \cdot \frac{QB}{QF} = 1.309 \times \frac{0.045}{0.06} = 0.982 \text{ m/s}$$

v_b can also be obtained graphically as follows:

Take vector $\mathbf{v}_a$ to a convenient scale [Fig. 2.18(b)]. Produce **oa** to **e** such that **oe**/**oa** = OE/OA. Rotate **oe** to **of** so that **of** is perpendicular to QF. Mark the point **b** on **qf** such that **qb**/**qf** = QB/Qf.

Now, $\mathbf{v}_{co} = \mathbf{v}_{cq}$

$$\mathbf{v}_{ca} + \mathbf{v}_{ao} = \mathbf{v}_{cb} + \mathbf{v}_{bq}$$

or

$$\mathbf{v}_{ao} + \mathbf{v}_{ca} = \mathbf{v}_{bq} + \mathbf{v}_{cb}$$

or

$$\mathbf{oa} + \mathbf{ac} = \mathbf{qb} + \mathbf{bc}$$

$\mathbf{v}_{ao}$ and $\mathbf{v}_{bq}$ are already there in the velocity diagram.

$\mathbf{v}_{ca}$ is $\perp AC$, draw a line $\perp AC$ through **a**;

$\mathbf{v}_{cb}$ is $\perp BC$, draw a line $\perp BC$ through **b**;

Thus, the point **c** is located.

Further, $\mathbf{v}_{do} = \mathbf{v}_{dc} + \mathbf{v}_{co}$

or

$$\mathbf{v}_{dg} = \mathbf{v}_{co} + \mathbf{v}_{dc}$$

or

$$\mathbf{gd} = \mathbf{oc} + \mathbf{cd}$$

$\mathbf{v}_{co}$ already exists in the diagram.

$\mathbf{v}_{dc}$ is $\perp CD$, draw **cd** $\perp CD$ through **c**;

$\mathbf{v}_{dg}$ is horizontal. Draw a horizontal line through **g** (or **o**) and locate the point **d**.

(i) v_d = **od** = <u>0.34 m/s</u>

(ii) $\omega_{bc} = \frac{v_{bc}}{BC} = \frac{0.12}{0.185} = \underline{0.649 \text{ rad/s}}$ clockwise

$\omega_{cd} = \frac{v_{dc}}{DC} = \frac{1.0}{0.24} = \underline{4.17 \text{ rad/s}}$ counter-clockwise

(iii) $T.\omega = F_d . v_d$

$F_d = 3000$ N

v_d = **od** (=**gd**) = 0.34 m/s

$$T = \frac{3000 \times 0.34}{(2\pi \times 100)/60} = \underline{97.4 \text{ N.m}}$$

Example 2.11 *The mechanism of a stone-crusher is shown in Fig. 2.19a along with various dimensions of links in mm. If crank OA rotates at a uniform velocity of 120 rpm, determine the velocity of the point K (jaw) when the crank OA is inclined at an angle of 30° to the horizontal. What will be the torque required at the crank OA to overcome a horizontal force of 40 kN at K?*

Solution $\omega_{ao} = \frac{2\pi \times 120}{60} = 12.6$ rad/s

$v_{ao} = \omega_{ao} \times OA = 12.6 \times 0.1 = 1.26$ m/s

Write the vector equation for the mechanism $OABQ$ and complete the velocity diagram as usual [(Fig. 2.19(b)]. Make **Δbac** similar to ΔBAC (both are read clockwise).

Write the vector equation for the mechanism $OACDM$ and complete the velocity diagram. Make **Δdmk** similar to ΔDMK (both are read clockwise).

v_k = **ok** = 0.45 m/s

v_k (horizontal) = 0.39 m/s

$T.\omega_{ao} = F_k\, v_k$ (horizontal)

$$T = \frac{40\,000 \times 0.39}{12.6} = \underline{1242 \text{ N.m}}$$

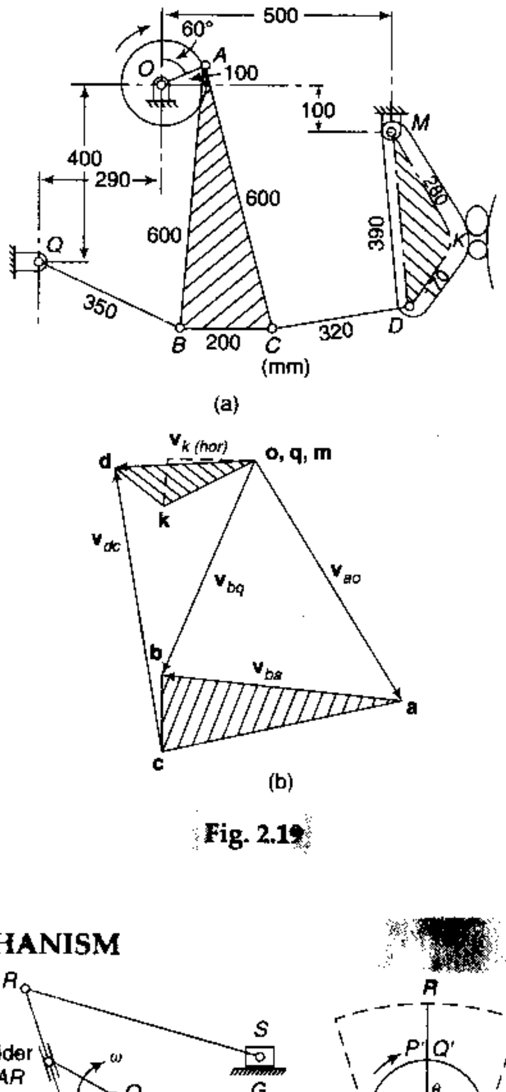

Fig. 2.19

2.10 CRANK- AND SLOTTED-LEVER MECHANISM

While analysing the motions of various links of a mechanism, sometimes we are faced with the problem of describing the motion of a moveable point on a link which has some angular velocity. For example, the motion of a slider on a rotating link. In such a case, the angular velocity of the rotating link along with the linear velocity of the slider may be known and it may be required to find the absolute velocity of the slider.

A crank and slotted-lever mechanism, which is a form of quick-return mechanism used for slotting and shaping machines, depicts the same form of motion [Fig. 2.20(a)]. *OP* is the crank rotating at an angular velocity of ω rad/s in the

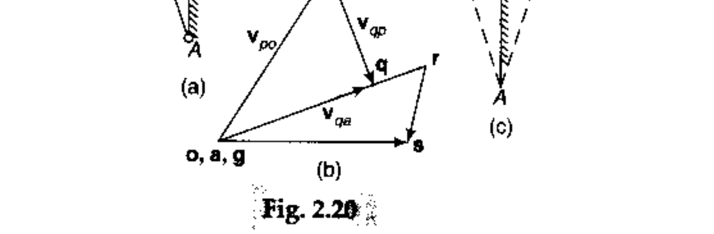

Fig. 2.20

clockwise direction about the centre *O*. At the end of the crank, a slider *P* is pivoted which moves on an oscillating link *AR*.

In such problems, it is convenient if a point *Q* on the link *AR* immediately below *P* is assumed to exist (*P* and *Q* are known as coincident points). As the crank rotates, there is relative movement of the points *P* and *Q* along *AR*.

Writing the vector equation for the mechanism *OPA*,

Vel. of *Q* rel. to *O* = Vel. of *Q* rel. to *P* + Vel. of *P* rel. to *O*

$$\mathbf{v}_{qo} = \mathbf{v}_{qp} + \mathbf{v}_{po}$$

or

$$\mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp}$$

or

$$\mathbf{aq} = \mathbf{op} + \mathbf{pq}$$

In this equation,

$\mathbf{v}_{po}$ or $\mathbf{op} = \omega.OP$; $\perp$ to *OP*

$\mathbf{v}_{qp}$ or **pq** is unknown in magnitude ; | | to *AR*

$\mathbf{v}_{qa}$ or **aq** is unknown in magnitude ; $\perp$ to *AR*

Take the vector $\mathbf{v}_{po}$ which is fully known [Fig. 2.20 (b)].

$\mathbf{v}_{qp}$ is | | *AR*, draw a line | | to *AR* through **p**;

$\mathbf{v}_{qa}$ is $\perp$ *AR*, draw a line $\perp$ *AR* through **a** (or **o**).

The intersection locates the point **q**.

The vector equation for the above could also have been written as

Vel. of *P* rel. to *A* = Vel. of *P* rel. to *Q* + Vel. of *Q* rel. to *A*

$$\mathbf{v}_{pa} = \mathbf{v}_{pq} + \mathbf{v}_{qa}$$

or

$$\mathbf{v}_{po} = \mathbf{v}_{qa} + \mathbf{v}_{pq}$$

or

$$\mathbf{op} = \mathbf{aq} + \mathbf{qp}$$

Take the vector $\mathbf{v}_{po}$ which is completely known.

$\mathbf{v}_{qa}$ is $\perp$*AR*, draw a line $\perp$*AR* through **a**;

$\mathbf{v}_{pq}$ is | | *AR*, draw a line | | *AR* through **p**.

The intersection locates the point **q**. Observe that the velocity diagrams obtained in the two cases are the same except that the direction of $\mathbf{v}_{pq}$ is the reverse of that of $\mathbf{v}_{qp}$.

As the vectors **oq** and **qp** are perpendicular to each other, the vector $\mathbf{v}_{po}$ may be assumed to have two components, one perpendicular to *AR* and the other parallel to *AR*.

The component of velocity along AR, i.e., **qp** indicates the relative velocity between Q and P or the velocity of sliding of the block on link AR.

Now, the velocity of R is perpendicular to AR. As the velocity of Q perpendicular to AR is known, the point **r** will lie on vector **aq** produced such that **ar**/**aq** = AR/AQ

To find the velocity of ram S, write the velocity vector equation,

$$\mathbf{v}_{so} = \mathbf{v}_{sr} + \mathbf{v}_{ro}$$

or

$$\mathbf{v}_{sg} = \mathbf{v}_{ro} + \mathbf{v}_{sr}$$

or

$$\mathbf{gs} = \mathbf{or} + \mathbf{rs}$$

$\mathbf{v}_{ro}$ is already there in the diagram. Draw a line through **r** perpendicular to RS for the vector $\mathbf{v}_{sr}$ and a line through **g**, parallel to the line of motion of the slider S on the guide G, for the vector $\mathbf{v}_{sg}$. In this way the point **s** is located.

The velocity of the ram S = **os** (or **gs**) towards right for the given position of the crank.

Also, $\omega_{rs} = \dfrac{v_{rs}}{RS}$ clockwise

Usually, the coupler RS is long and its obliquity is neglected.

Then **or** ≈ **os**

Referring Fig. 2.20 (c),

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$

When the crank assumes the position OP' during the cutting stroke, the component of velocity along AR (i.e., **pq**) is zero and **oq** is maximum (= **op**)

Let r = length of crank (= OP)

l = length of slotted lever (= AR)

c = distance between fixed centres (= AO)

ω = angular velocity of the crank

Then, during the cutting stroke,

$$v_{s\,\max} = \omega \times OP' \times \frac{AR}{AQ} = \omega r \times \frac{l}{c+r}$$

This is by neglecting the obliquity of the link RS, i.e. assuming the velocity of S equal to that of R.

Similarly, during the return stroke,

$$v_{s\,\max} = \omega \times OP'' \times \frac{AR}{AQ''} = \omega r \times \frac{l}{c-r}$$

$$\frac{v_{s\,\max}(\text{cutting})}{v_{s\,\max}(\text{return})} = \frac{\omega r \dfrac{l}{c+r}}{\omega r \dfrac{l}{c-r}} = \frac{c-r}{c+r}$$

Example 2.12 *Figure 2.21(a) shows the link mechanism of a quick return mechanism of the slotted lever type, the various dimensions of which are*

OA = 400 mm, OP = 200 mm, AR = 700 mm, RS = 300 mm

For the configuration shown, determine the velocity of the cutting tool at S and the angular velocity of the link RS. The crank OP rotates at 210 rpm.

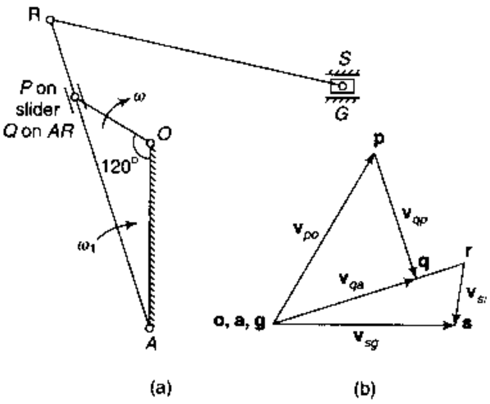

Fig. 2.21

Solution $\omega_{po} = \dfrac{2\pi \times 210}{60} = 22$ rad/s

Draw the configuration to a suitable scale. The vector equation for the mechanism *OPA*,

$$\mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp} \quad \text{or} \quad \mathbf{aq} = \mathbf{op} + \mathbf{pq}$$

In this equation,

$$\mathbf{v}_{po} \text{ or } \mathbf{op} = \omega . OP = 22 \times 0.2 = 4.4 \text{ m/s}$$

Take the vector $\mathbf{v}_{po}$ which is fully known [Fig. 2.21(b)].

$\mathbf{v}_{qp}$ is || *AR*, draw a line || to *AR* through **p**;

$\mathbf{v}_{qa}$ is ⊥ *AR*, draw a line ⊥ *AR* through **a** (or **o**).

The intersection locates the point **q**. Locate the point **r** on the vector **aq** produced such that **ar**/**aq** = *AR*/*AQ*.

Draw a line through **r** perpendicular to *RS* for the vector $\mathbf{v}_{sr}$ and a line through **g**, parallel to the line of motion of the slider *S* on the guide *G*, for the vector $\mathbf{v}_{sg}$. In this way the point **s** is located.

The velocity of the ram *S* = **os** (or **gs**) = 4.5 m/s

It is towards right for the given position of the crank.

Angular velocity of link *RS*,

$$\omega_{rs} = \frac{v_{rs}}{RS} = \frac{1.4}{0.3} = 4.67 \text{ rad/s clockwise}$$

Example 2.13 *For the inverted slider-crank mechanism shown in Fig. 2.22(a), find the angular velocity of the link QR and the sliding velocity of the block on the link QR. The crank OA is 300 mm long and rotates at 20 rad/s in the clockwise direction. OQ is 650 mm and QOA = 40°*

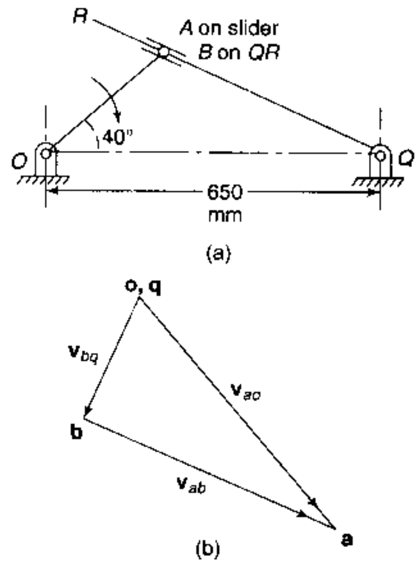

Fig. 2.22

Solution The velocity vector equation can be written as usual.

$\mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$	or	$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$
$\mathbf{v}_{ao} = \mathbf{v}_{bq} + \mathbf{v}_{ab}$		$\mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$
oa = qb + ba		**qb = oa + ab**

$\mathbf{v}_{ao}$ is fully known and after taking this vector, draw lines for $\mathbf{v}_{bq}$ and $\mathbf{v}_{ab}$ (or $\mathbf{v}_{ba}$) and locate the point **b**. obviously, the direction-sense of $\mathbf{v}_{ab}$ is opposite to that of $\mathbf{v}_{ba}$. Figure 2.22 (b) shows the solution of the first equation.

$$\omega_{qr} = \omega_{qb} = \frac{v_{qb} \text{ or } v_{bq}}{BQ}$$

$$= \frac{2.55}{0.46} \qquad (BQ = 0.46 \text{ m on measuring})$$

$$= \underline{5.54 \text{ rad/s counter - clockwise}}$$

Sliding velocity of block = v_{ba} or **ab** = 5.45 m/s

Example 2.14 *For the position of the mechanism shown in Fig. 2.23(a), calculate the angular velocity of the link AR. OA is 300 mm long and rotates at 20 rad/s in the clockwise direction. OQ = 650 mm and QOA = 40°*

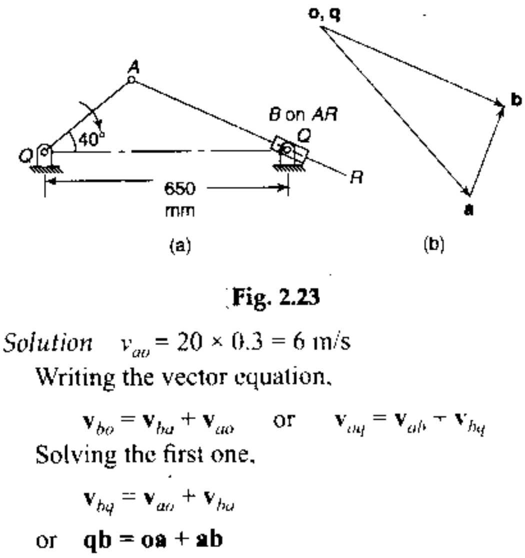

Fig. 2.23

Solution $v_{ao} = 20 \times 0.3 = 6$ m/s

Writing the vector equation,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao} \quad \text{or} \quad \mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

Solving the first one,

$$\mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or **qb = oa + ab**

Take $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.23(b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a**;

$\mathbf{v}_{bq}$ is along AB, draw a line || to AB through **q**.

The intersection locates the point **b**.

$$\omega_{ar} = \omega_{ab} = \frac{v_{ab} \text{ or } v_{ba}}{AB} = \frac{2.55}{0.46}$$

$= \underline{5.54 \text{ rad/s}}$ counter–clockwise

Example 2.15 *In the pump mechanism shown in Fig. 2.24(a), OA = 320 mm, AC = 680 mm and OQ = 650 mm. For the given configuration, determine the*

(i) angular velocity of the cylinder

(ii) sliding velocity of the plunger

(iii) absolute velocity of the plunger

The crank OA rotates at 20 rad/s clockwise.

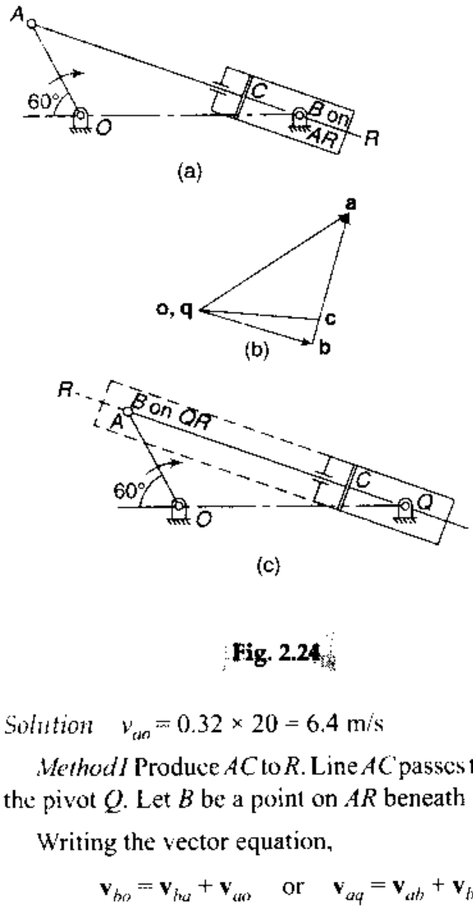

Fig. 2.24

Solution $v_{ao} = 0.32 \times 20 = 6.4$ m/s

Method I Produce AC to R. Line AC passes through the pivot Q. Let B be a point on AR beneath Q.

Writing the vector equation,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao} \quad \text{or} \quad \mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

Solving any of these equations leads to same velocity diagram except for the direction of $\mathbf{v}_{ba}$ and $\mathbf{v}_{ab}$.

Taking the latter equation,

$$\mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

or

$$\mathbf{v}_{ao} = \mathbf{v}_{bq} + \mathbf{v}_{ab}$$

or $\mathbf{oa} = \mathbf{qb} + \mathbf{ba}$

Complete the velocity triangle as usual [Fig. 2.24(b)]

Locate point **c** on **ab** such that $\dfrac{\mathbf{ac}}{\mathbf{ab}} = \dfrac{AC}{AB}$

(i) Angular velocity of cylinder = Angular velocity of AR or AB

$$= \frac{v_{ab}}{AB}$$

$$= \frac{4.77}{0.85} = \underline{5.61 \text{ rad/s}} \text{ clockwise}$$

(ii) Sliding velocity of plunger = Velocity of B relative to Q
$= \mathbf{qb} = \underline{4.1 \text{ m/s}}$

(iii) Absolute velocity of plunger = **oc** or **qc** = $\underline{4.22 \text{ m/s}}$

Method II Link AC is integrated with the plunger and thus A can be considered to be a point on it. Assume the cylinder to be of such a length that a point B is located on it just beneath the point A. [Fig. 2.24 (c)].

Writing the vector equation,

$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$ or $\mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$

$\mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$ $\mathbf{v}_{ao} = \mathbf{v}_{bq} + \mathbf{v}_{ab}$

$\mathbf{qb} = \mathbf{oa} + \mathbf{ab}$ $\mathbf{oa} = \mathbf{qb} + \mathbf{ba}$

Thus, the same equations have been obtained as in Method-I and thus can be solved easily.

Example 2.16 *A Whitworth quick-return mechanism has been shown in Fig. 2.25(a). The dimensions of the links are OP (crank) = 240 mm, OA = 150 mm, AR = 165 mm and RS = 430 mm. The crank rotates at an angular velocity of 2.5 rad/s. At the moment when the crank makes an angle of 45° with the vertical, calculate the*

(i) velocity of the ram S

(ii) velocity of the slider P on the slotted lever

(iii) angular velocity of the link RS

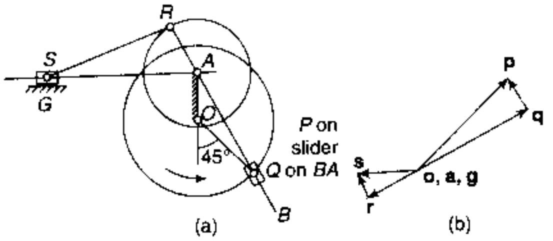

Fig. 2.25

Solution $v_p = 2.5 \times 0.24 = 0.6$ m/s

Locate a point Q on AB beneath point P on the slider.

Solve any of the following velocity vector equations,

$\mathbf{v}_{pa} = \mathbf{v}_{pq} + \mathbf{v}_{qa}$ or $\mathbf{v}_{qa} = \mathbf{v}_{qp} + \mathbf{v}_{pa}$

Produce **qa** to **r** such that $\dfrac{\mathbf{ar}}{\mathbf{qa}} = \dfrac{AR}{QA}$ [Fig. 2.25(b)]

Now, $\mathbf{v}_{sa} = \mathbf{v}_{sr} + \mathbf{v}_{ra}$

Complete the velocity diagram as indicated by this equation

(i) $v_s = \mathbf{gs} = \underline{0.276 \text{ m/s}}$

(ii) $v_{pq} = \mathbf{qp} = \underline{0.177 \text{ m/s}}$

(iii) $\omega_{rs} = \dfrac{v_{rs} \text{ or } v_{sr}}{RS} = \dfrac{0.12}{0.43} =$ $\underline{0.279 \text{ rad/s}}$ clockwise

Example 2.17 *In the mechanism shown in Fig. 2.26(a), the crank OP rotates at 210 rpm in the counter-clockwise direction and imparts vertical reciprocating motion to rack through a toothed quadrant. Slotted bar and the quadrant oscillate about the fixed pivot A. Determine for the given position the*

(i) linear speed of the rack

(ii) ratio of the times of raising and lowering of the rack

(iii) stroke of the rack

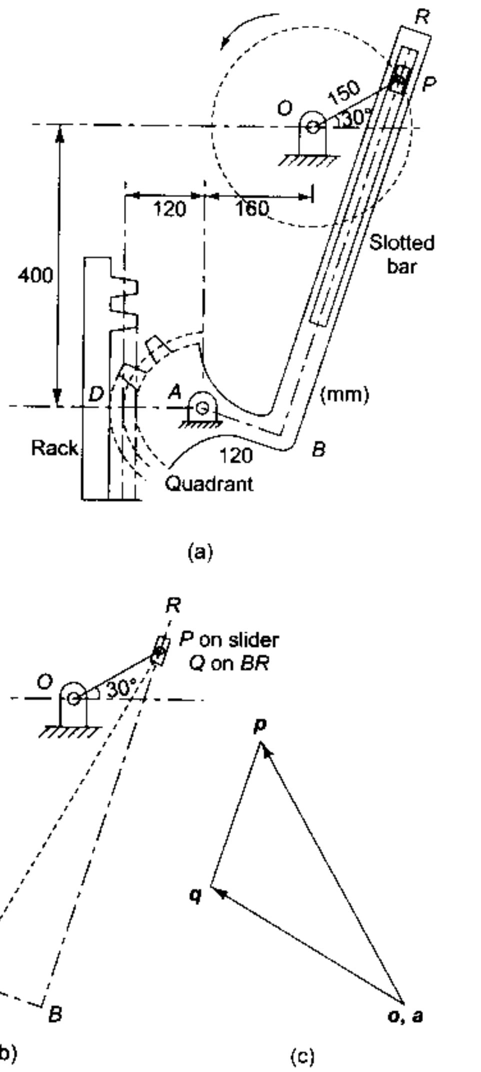

Fig. 2.26

Solution $\omega_{po} = \frac{2\pi N}{60} = \frac{2\pi \times 210}{60} = 22$ rad/s

$v_{po} = 22 \times 0.15 = 3.3$ m/s

Draw the configuration diagram to a suitable scale [Fig. 2.26(b)].

Locate a point Q on BR beneath point P on the slider.

Then the vector equation is

$$\mathbf{v}_{qo} = \mathbf{v}_{qp} + \mathbf{v}_{po} \text{ or } \mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp}$$

Take the vector $\mathbf{v}_{po}$ to a convenient scale in the proper direction and sense [Fig. 2.26(c)].

$\mathbf{v}_{qp}$ is along BR, draw a line parallel to BR through **p**;

Now, Q is a point on the link ABR which is pivoted at point A. The direction of velocity of any point on the link is perpendicular to the line joining that point with the pivoted point A.

$\mathbf{v}_{qa}$ is $\perp QA$, draw a line $\perp QA$ through **a**;

The intersection of the two lines locates the point **q**.

Now angular velocity of the quadrant and the lever ABQ,

$$\omega_{aq} = \frac{v_{aq}}{AQ} = \frac{2.5}{0.577} = 4.33 \text{ rad/s}$$

counter-clockwise

(i) The linear velocity of the rack will be equal to the tangential velocity of the quadrant at the teeth, i.e.,

$v_r = \omega \times AD = \omega \times 120 = 4.33 \times 120 =$ 519.6 mm/s

(ii) The reciprocating rack changes the direction when the crank OP assumes a position such that the tangent at P to the circle at O is also a tangent to the circle at A with radius AB as shown in Fig. 2.27. The rack is lowered during the rotation of the crank from P to P' and is raised when P' moves to P counter-clockwise.

Thus,

$$\frac{\text{Time of lowering}}{\text{Time of raising}} = \frac{\theta}{\beta} = \frac{215°}{135°} = 1.59$$

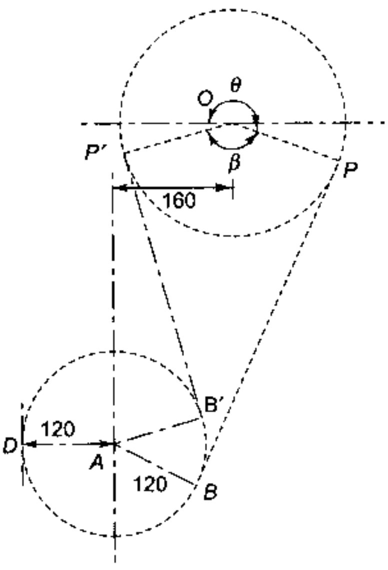

Fig. 2.27

(iii) Stroke of the rack = angular displacement of the quadrant × its radius

= angle $BAB' \times AB$

$= 44 \times \frac{\pi}{180} \times 120 = 92.2$ mm

($\angle BAB' = 44°$ by measurement)

Example 2.18 *In the swiveling-joint mechanism shown in Fig. 2.28(a), AB is the driving crank rotating at 300 rpm clockwise.*

The lengths of the various links are

AD = 650 mm, AB = 100 mm, BC = 800 mm, DC = 250 mm, BE = CE, EF = 400 mm, OF = 240 mm, FS = 400 mm

For the given configuration of the mechanism, determine the

(i) velocity of the slider block S

(ii) angular velocity of the link EF

(iii) velocity of the link EF in the swivel block

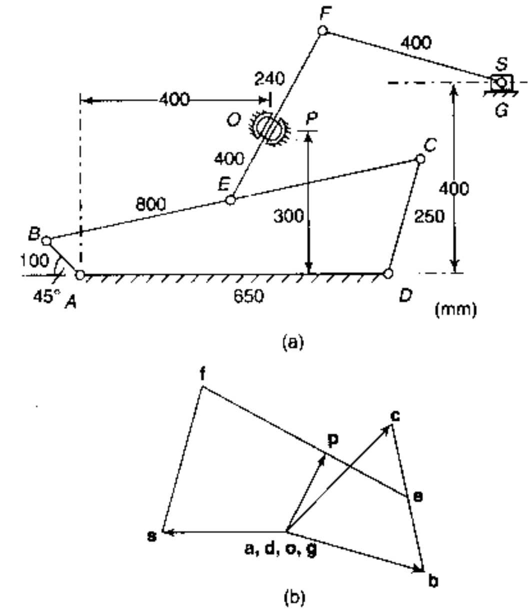

Fig. 2.28

Solution $\omega_{ba} = \frac{2\pi \times 300}{60} = 31.4$ rad/s

$v_b = 31.4 \times 0.1 = 3.14$ m/s

The velocity diagram is completed as follows:

- Draw the velocity diagram of the four-link mechanism *ABCD* as usual starting with the vector **ab** as shown in Fig. 2.28(b).
- Locate the point **e** in the velocity diagram at the midpoint of **bc** as the point *E* is the midpoint of *BC*. Let *Q* be a point on the link *EF* at the joint *O*. Draw a line ⊥ *EQ* through **e**, a point on which will represent the velocity of *Q* relative to *E*.
- The sliding velocity of link *EF* in the joint at the instant is along the link. Draw a line parallel to *EF* through **o**, the intersection of which with the previous line locates the point **q**.
- Extend the vector **eq** to **f** such that **ef/eq** = *EF/EQ*.

- Through **f** draw a line ⊥ FS, and through **g** a line parallel to line of stroke of the slider. The intersection of the two lines locates the point **s**.

Thus, the velocity diagram is completed.

(i) The velocity of slider S = **gs** = 2.6 m/s

(ii) The angular velocity of the link EF

$$= \frac{v_{fe}}{EF} = \frac{\mathbf{ef}}{EF} = \frac{4.9}{0.4} = 12.25 \text{ rad/s (ccw)}$$

(iii) The velocity of the link EF in the swivel block = **oq** = 1.85 m/s

2.11 ALGEBRAIC METHODS

Vector Approach

In Sec. 2.10, the concept of coincident points was introduced. However, complex algebraic methods provide an alternative formulation for the kinematic problems. This also furnishes an excellent means of obtaining still more insight into the meaning of the term *coincident points*.

Let there be a plane moving body having its motion relative to a fixed coordinate system xyz (Fig. 2.29). Also, let a moving coordinate system $x'y'z'$ be attached to this moving body. Coordinates of the origin A of the moving system are known relative to the absolute reference system. Assume that the moving system has an angular velocity ω also.

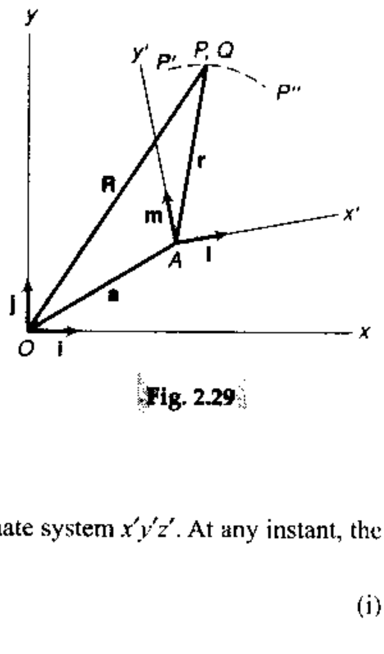

Fig. 2.29

Let

i,j,k unit vectors for the absolute system
l,m,n unit vectors for the moving system
ω angular velocity of rotation of the moving system
R vector relative to fixed system
r vector relative to moving system

Let a point P move along path $P'PP''$ relative to the moving coordinate system $x'y'z'$. At any instant, the position of P relative to the fixed system is given by the equation

$$\mathbf{R} = \mathbf{a} + \mathbf{r} \tag{i}$$

where $\mathbf{r} = x'\mathbf{l} + y'\mathbf{m} + z'\mathbf{n}$

Thus, (i) may be written as, $\mathbf{R} = \mathbf{a} + x'\mathbf{l} + y'\mathbf{m} + z'\mathbf{n}$

Taking the derivatives with respect to time to find the velocity,

$$\dot{\mathbf{R}} = \dot{\mathbf{a}} + (\dot{x}'\mathbf{l} + \dot{y}'\mathbf{m} + \dot{z}'\mathbf{n}) + (x'\dot{\mathbf{l}} + y'\dot{\mathbf{m}} + z'\dot{\mathbf{n}}) \tag{ii}$$

The first term in this equation indicates the velocity of the origin of the moving system. The second term refers to the velocity of P relative to the moving system. The third term is due to the fact that the reference system has also rotary motion with angular velocity ω.

Also, $\dot{\mathbf{l}} = \omega \times \mathbf{l}$, $\dot{\mathbf{m}} = \omega \times \mathbf{m}$, $\dot{\mathbf{n}} = \omega \times \mathbf{n}$

Therefore, Equation (ii) becomes

$$\dot{\mathbf{R}} = \dot{\mathbf{a}} + (\dot{x}'\mathbf{l} + \dot{y}'\mathbf{m} + \dot{z}'\mathbf{n}) + \omega(x'\mathbf{l} + y'\mathbf{m} + z'\mathbf{n}$$

The above equation can be written in the form,

$$\mathbf{v}_p = \mathbf{v}_a + \mathbf{v}^R + \omega \times \mathbf{r} \tag{2.7}$$

The second term known as *relative velocity* is the velocity which an observer attached to the moving system would report for point P, i.e., velocity of P relative to the moving body or system. This also implies that it is the velocity relative to a coincident point Q on the moving body since the observer may be stationed at the point Q on the moving body.

Now, the absolute velocity of the coincident point Q on the moving system which coincides with the point P at the instant may be written as

$$\begin{aligned} \mathbf{v}_{qo} &= \mathbf{v}_{qa} + \mathbf{v}_{ao} \\ &= \mathbf{v}_{ao} + \mathbf{v}_{qa} \\ &= \mathbf{v}_a + \omega \times \mathbf{r} \end{aligned}$$

and equation (iii) changes to $\mathbf{v}_p = \mathbf{v}_{qo} + \mathbf{v}^R$

Thus absolute velocity of the point P moving relative to a moving reference system is equal to the velocity of the point relative to the moving system plus the absolute velocity of a coincident point fixed to the moving reference system.

The above equation may be written as

$$\begin{aligned} \mathbf{v}_{po} &= \mathbf{v}_{qo} + \mathbf{v}_{pq} \\ \mathbf{v}_{po} &= \mathbf{v}_{pq} + \mathbf{v}_{qo} \end{aligned}$$

Vel. of P rel. to O = Vel. of P rel. to Q + Vel. of Q rel. to O

Use of Complex Numbers

In a complex number system, a vector connecting two points O and P (Fig. 2.30) may be expressed as

$\mathbf{r} = a + ib$ in the rectangular form

where a and b are known as *real* and *imaginary* parts of $\mathbf{r}$. The real part is always taken along the X-axis from the origin, to the right if positive and to the left if negative. The symbol i prefixed to b indicates that it is to be taken at an angle of 90° in the counter-clockwise direction from the positive x-direction.

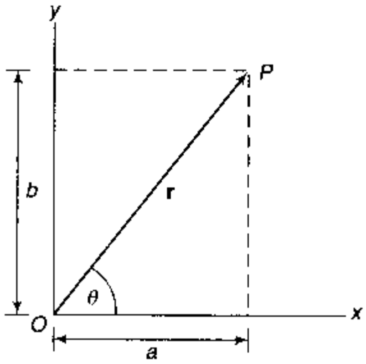

Fig. 2.30

As $i = \sqrt{-1}$ indicates 90° counter-clockwise direction,

Therefore,

$i^2 = (\sqrt{-1})^2 = -1$ is 180° counter-clockwise

$i^3 = (\sqrt{-1})^3 = (\sqrt{-1})^2\sqrt{-1} = (-1)i = -i$ is 270° counter-clockwise or 90° clockwise

In the polar form **r** can be expressed as

$$\mathbf{r} = r\cos\theta + ir\sin\theta = r(\cos\theta + i\sin\theta)$$

In this equation r is said to be magnitude of **r**, denoted by $|\mathbf{r}|$ and θ is called the argument of **r**, denoted by arg(**r**).

Since r is the magnitude of vector **r**, the term in the parenthesis in the above equation plays the role of a unit vector which points in the direction of OP.

From trigonometry, it can be written that

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Therefore,

$$\mathbf{r} = re^{i\theta} \text{ which is the complex polar form.} \quad \text{(i)}$$

Complex numbers are assumed to follow all the formal rules of real algebra.

Velocity

Differentiating Eq. (i) with respect to time,

$$\mathbf{v} = \dot{r}e^{i\theta} + ir\dot{\theta}e^{i\theta}$$
$$= (\dot{r} + ir\dot{\theta})e^{i\theta} \quad (2.8)$$

2.12 INSTANTANEOUS CENTRE (I-CENTRE)

Let there be a plane body p having a non-linear motion relative to another plane body q. At any instant, the linear velocities of two points A and B on the body p are $\mathbf{v}_a$ and $\mathbf{v}_b$ respectively in the directions as shown in Fig. 2.31.

If a line is drawn perpendicular to the direction of $\mathbf{v}_a$ at A, the body can be imagined to rotate about some point on this line. Similarly, the centre of rotation of the body also lies on a line perpendicular to the direction of $\mathbf{v}_b$ at B. If the intersection of the two lines is at I, the body p will be rotating about I at the instant. This point I is known as the *instantaneous centre of velocity* or more commonly *instantaneous centre of rotation* for the body p. This property is true only for an instant and a new point will become the instantaneous centre at the next instant. Thus, it is a misnomer to call this point the centre of rotation, as generally this point is not located at the centre of curvature of the apparent path taken by a point

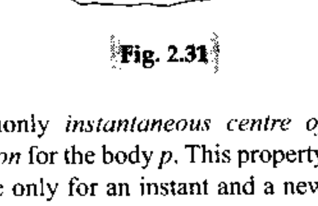

Fig. 2.31

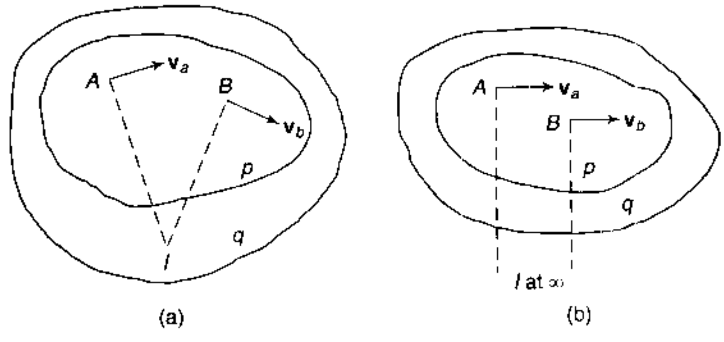

Fig. 2.32